THREE ESSAYS ON HOUSEHOLD HETEROGENEITY AND MACROECONOMIC DYNAMICS

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Abstract

This thesis consists of three essays on the implications of household heterogeneity for macroeconomic dynamics, both in the short and the long run. Each essay employs a different Heterogeneous-Agent Dynamic Stochastic General-Equilibrium (DSGE) model tailored to the given research question. The type of heterogeneity varies across models, but they all have in common that the marginal propensity to consume (MPC) is heterogeneous and depends negatively on household wealth. These stylized facts have been shown to be crucial for many research questions, yet they cannot be captured in traditional Representative-Agent models.

The first essay computes multipliers of different types of budget-neutral redistributive fiscal policies in a New Keynesian DSGE model. An ad-hoc distribution of MPC is introduced by partitioning the household population into a large number of segments with a varying share of hand-to-mouth consumers. A sufficiently granular segmentation allows to match empirical estimates of the MPC distribution, which has been a challenge in previous theoretical contributions. I find that targeted transfers can be an effective tool in stimulating aggregate demand, in particular when they redistribute funds from the very top to the bottom of the MPC distribution. This finding is particularly relevant for countries with limited fiscal space.

In the second essay, I analyze the role of household heterogeneity for the propagation of external shocks in a Bewley-type model of a small open economy. As markets are incomplete and households are hit by idiosyncratic shocks, households self-insure by holding different levels of precautionary savings in the form of an asset denominated in foreign currency. I find that negative external shocks reduce households' current income, whereby poor households are affected most strongly. Since poor households have the highest MPC, this brings about a reduction in aggregate demand. My results further show that rich households dis-proportionally benefit from the stabilization of the domestic economy, provided by a devaluation of the nominal exchange rate. In an extension, Stone-Geary preferences are introduced to generate relatively larger tradable expenditure shares for poor households, as empirically observed. I show that this assumption generates a friction that prevents poorer households from switching from tradable to non-tradable consumption in response to an adverse shock, which amplifies aggregate crisis dynamics.

While the first two essays focus on short-run dynamics, in the third essay Philipp Engler and I take on a long-run perspective in analyzing the effect of an increase in life expectancy in an Aiyagari overlapping generations model. Motivated by empirical evidence, we modify the process for idiosyncratic wage shocks such that their volatility is u-shaped over the life cycle. Relative to the standard model with age-independent wage volatility, we find a more significant role of labor supply adjustment in preparing for an increase in the expected retirement spell, while precautionary savings become less relevant. In the aggregate, this translates into a smaller fall in the natural interest rate, relative to the standard model, and raising the retirement age to limit the decrease of the natural rate proves less effective.

Zusammenfassung

Diese Dissertation besteht aus drei Essays, die die Rolle von Haushalts-Heterogenität für kurz- und langfristige makroökonomische Entwicklungen untersuchen. Alle Essays verwenden dynamische, stochastische Modelle des allgemeinen makroökonomischen Gleichgewichtes, in denen Haushalte heterogen sind. Die genauen Modellierungsansätze sind auf die Forschungsfrage des jeweiligen Essays zugeschnitten. Die Modelle eint, dass sie Heterogenität in der marginalen Konsumneigung abbilden, wobei reichere Haushalte eine geringere Konsumneigung haben. Dieser Aspekt ist entscheidend für viele Forschungsfragen, wird jedoch in Modellen mit einem repräsentativen Haushalt vernachlässigt.

Das erste Essay berechnet Multiplikatoren von budgetneutralen fiskalischen Politiken, die Einkommen von Haushalten mit einer geringen Konsumneigung zu Haushalten mit einer hohen Konsumneigung umverteilen. Zu diesem Zweck unterteile ich den Haushaltssektor in einem Neu-Keynesianischen Modell in eine Vielzahl von Untergruppen mit unterschiedlichen Konsumneigungen, wodurch die empirisch beobachtete Verteilung von Konsumneigungen exakt im Modell repliziert werden kann. Budgetneutrale Umverteilungen sind in diesem Modell ein wirksames Instrument um die Wirtschaft zu stimulieren, besonders wenn der Fokus auf Gruppen mit besonders hoher bzw. besonders niedriger Konsumneigung liegt. Dieses Resultat ist besonders wichtig wenn der fiskalische Spielraum gering ist.

Das zweite Essay analysiert die Auswirkungen von Haushalts-Heterogenität für die Transmission von adversen externen Schocks in einem Bewley-Modell einer kleinen offenen Volkswirtschaft. Da der Anlagemarkt unvollständig ist und Haushalte idiosynkratischen Schocks ausgesetzt sind, häufen sie unterschiedliche Vermögen in ausländischer Währung an. Ich zeige, dass die negativen Auswirkungen der Schocks auf Haushaltseinkommen für ärmere Haushalte stärker sind. Da diese eine höhere marginale Konsumneigung haben, bedeutet das eine Abnahme der aggregierten Nachfrage. Des Weiteren profitieren reichere Haushalte disproportional von einer Stabilisierung der Volkswirtschaft durch eine Wechselkursabwertung. In einer Erweiterung berücksichtige ich zusätzlich, dass ärmere Haushalte relativ mehr handelbare Güter konsumieren. Dies erzeugt eine Friktion, die diese Haushalte davon ab-hält handelbare durch nicht-handelbare Güter zu substituieren, was die Krise verschlimmert.

Während sich die ersten beiden Essays mit der kurzen Frist befassen, untersuchen Philipp Engler und ich im dritten Essay die langfristigen Auswirkungen eines Anstieges der Lebenserwartung. Dabei setzen wir den Fokus auf die Implikationen von altersabhängigem Lohnrisiko, das, wie in empirischen Studien belegt, einen U-förmigen Verlauf über die Lebenszeit aufweist. Wenn diese Charakteristik in einem Aiyagari-Modell mit überlappenden Generationen berücksichtigt wird, spielt die Anpassung von Arbeitsangebot in Vorbereitung auf eine längere Lebenserwartung eine wichtigere Rolle als im Standard-Modell ohne altersabhängiges Lohnrisiko. Anpassungen im Sparverhalten werden hingegen weniger wichtig. Im Aggregat bedeutet dies, dass der gleichgewichtige Zins in Reaktion auf eine Erhöhung der Lebenserwartung weniger stark fällt als im Standard-Modell. Außerdem ist eine Erhöhung des Renteneintrittsalters weniger wirksam um dem Fall des gleichgewichtigen Zinses entgegen zu wirken.

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Chapter 1

Introduction

Despite dating back to the 1950s, the neoclassical growth model still constitutes the backbone of most contemporaneous theoretical macroeconomic models. It features a complete set of state-contingent bonds, which allows households to smooth consumption and the modeller to represent them by a single agent. What matters for consumption of this representative agent (RA) is her permanent income, which is little affected by temporary changes in current income. While various frictions have been introduced to strengthen model's ability fit the data, until recently the representative, permanent-income consumer remained prevalent in the analysis of short-term fluctuations and long-term growth. This thesis contributes to a growing literature that calls into question the representability of household behavior by a single agent and explores the role of household heterogeneity for aggregate phenomena.

The causes and consequences of income and wealth heterogeneity have been studied extensively in a cross-sectional context, e.g., in the literature on taxation. The dominant modelling strategy is based on Bewley (1977)'s incomplete-markets model, in which households have only access to a non-state-contingent bond and face a borrowing constraint. Idiosyncratic shocks cause households to accumulate different amounts of precautionary savings to self-insure, which gives rise to an endogenous wealth distribution and thereby heterogeneity in marginal propensities to consume (MPC).[1] Krusell and Smith (1998) were first to accomplish the computationally challenging task of integrating such an incomplete-markets setup into a business-cycle model. They found that time series generated by their heterogeneous-agent (HA) model were very similar to those obtained from a model version with a RA, which at first discouraged further research in this field. At the same time, however, there was a growing empirical literature initiated by Deaton (1992) and Campbell and Mankiw (1989) arguing that aggregate consumption strongly depends on current income. Those findings called into question the permanent-income hypothesis and sparked a different approach of modelling household heterogeneity. So-called "hand-to-mouth" (HtM) consumers – who consume their full disposable income in every period (see, e.g., Galí et al., 2007) – were modelled alongside Ricardian consumers (also labelled "two-agent" (TA) models). Albeit ad hoc, the approach proved successful in explaining consumption dynamics.

Several developments around the time of the Great Financial Crisis have revived interest in a micro-foundation of household heterogeneity. First, the crisis sparked research on the aggregate consequences of all kinds of financial frictions. While the main focus was on frictions in financial intermediation, also borrowing constraints of individual countries (Mendoza, 2010) and households (Oh and Reis, 2012)

[1] See also seminal papers by Imrohoroğlu (1989), Huggett (1993), and Aiyagari (1994).

attracted more attention. Second, a growing literature using micro data added to the existing macroeconomic evidence that consumption is in fact very sensitive to changes in current income (see Jappelli and Pistaferri, 2010, for an overview). Third, new and more accurate solution methods (e.g., Reiter, 2009) showed that Krusell and Smith (1998)'s result was by no means universal: Whether or not the transmission mechanism of an aggregate shock can be well approximated by a RA model depends on the nature of the shock and on the type of household heterogeneity.

This thesis contributes to answering several macroeconomic questions that either cannot be answered by the means of a RA model, or have only been answered in HA models that neglected potentially relevant dimensions of heterogeneity. The approach to model household heterogeneity is different in each of the chapters and is tailored to the specific research question. Both the existence of precautionary savings (as well as their cyclicality) and the heterogeneity in MPC have been shown to matter for the transmission of aggregate shocks (see Kaplan and Violante, 2018, for an overview). TA models abstract from precautionary savings but are able to capture heterogeneous responses to changes in current income, which is the most important aspect for many questions, as e.g. argued by Debortoli and Galí (2017) for monetary policy shocks. For other research questions, however, this modelling approach might be ill-suited. For example, a TA model fails to capture heterogeneous revaluation effects across households with different asset holdings. Acharya and Dogra (2020) find that, even absent MPC heterogeneity, precautionary savings can have an important role in shock propagation. Thus, it is not trivial to judge whether ad-hoc modeling of heterogeneity allows for a meaningful analysis of a given research question or if a fully-fledged incomplete-markets HA model is required.

The second chapter of this thesis "Redistribution Multipliers and the Distribution of Marginal Propensities to Consume" computes multipliers of a budget-neutral lump-sum redistribution from low-MPC households to high-MPC households. An empirically motivated ad-hoc distribution of MPC is introduced into a standard New Keynesian (NK) model by partitioning the population into sufficiently many household segments that vary in the share of hand-to-mouth consumers. This is in the tradition of TANK models, but allowing for a higher number of segments refines the modelling of the MPC distribution, which is arguably the most important determinant for redistribution multipliers. For this purpose, the ad-hoc approach is preferable over an incomplete-markets HANK model, as matching an empirical MPC distribution is a challenge in this model class. The model is solved by a first-order perturbation around its steady state. I find that targeted transfers can be an effective means to stimulate aggregate demand, in particular when they redistribute funds from the very top to the bottom of the MPC distribution. The multiplier is shown to be 0.14 for a redistribution of 1% of steady-state output from the top half of the MPC distribution to the bottom half, going up to 0.26 for a redistribution from the top 10% to the bottom 10%. This finding is particularly relevant for countries with limited fiscal space.

In the third chapter of the thesis, titled "Household Heterogeneity and the Adjustment to External Shocks of a Small Open Econonomy", I analyze the propagation of external shocks in a fully-fledged incomplete-markets HA model of a small open economy (SOE). Previous work on this topic has largely focused on the RA framework, thereby ignoring implied redistributional effects that, together with heterogeneous MPC, can affect aggregate dynamics. As my focus is on differentiated shock implications for households with different income levels, the choice of the Bewley (1977) setup, rather than ad-hoc heterogeneity, is a natural one. The model is a HA version of Schmitt-Grohé and Uribe (2016): Households consume tradable and non-tradable goods and hold different amounts of a single asset denominated in foreign currency to self-insure against idiosyncratic shocks. They receive income from supplying labor to firms that produce non-tradable goods and whose wage setting decision is subject to downward nominal wage rigidity. Michael Reiter's HetSol Toolkit, combining the methods in Reiter (2009) and Reiter (2010), allows to find a solution of the model that is fully non-linear in its idiosyncratic components but linear in aggregate shocks. I analyze the propagation of two adverse external shocks, a hike in the world interest rate and a terms-of-trade deterioration. Relative to the RA model, the adverse shocks mainly transmit via negative indirect effects on households' real income, rather than through direct effects on intertemporal substitution. Thereby, income of poor, low-MPC households is more adversely affected, forcing them to sharply decrease consumption. Comparing outcomes under a fixed and a flexible exchange-rate regime shows that high-income consumers dis-proportionally benefit from the stabilization provided by an exchange rate devaluation. In an extension, I introduce Stone-Geary preferences to capture empirical evidence showing that low-income households consume relatively more tradable goods. I find that this assumption generates a friction that prevents poorer households from switching from tradable to non-tradable consumption in response to an adverse shock, which amplifies crisis dynamics.

The last chapter "Preparing for Longer Lives with Age-Dependent Income Uncertainty" is co-authored with Philipp Engler. In contrast to the previous chapters it takes a long-run perspective and analyzes the effects of a permanent increase in life expectancy, with a novel focus on age-dependency of wage volatility. In particular, we incorporate a u-shaped life-time profile of wage risk – that has been documented in empirical studies – in an overlapping-generations Aiyagari (1994) model. To this end, the method to approximate the idiosyncratic wage process is modified to allow for age-dependent transition probabilities as proposed by Fella et al. (2019). For the research question at hand, an incomplete asset market is crucial to generate the link between age and wage risk, and eventually the relation between age and wealth holdings. The model is solved for an initial steady state and for the terminal steady state in which life expectancy is higher. We find that, relative to the standard model with age-independent wage risk (see Conesa et al., 1999), adjustments in labor supply play a greater role in preparing for a longer life, while adjustments in savings are relatively less important. In the aggregate this dampens the adjustment of the natural interest rate to a rise in life expectancy: the interest rate decline in response to a 7-year rise in life expectancy (as forecasted for the US from 2000 until 2050) is reduced by ten basis points. Increasing the retirement age to disincentive precautionary savings and thereby cushion the fall in the natural interest rate proves less effective than in the standard model.

Chapter 2

Redistribution Multipliers and the Distribution of Marginal Propensities to Consume

Abstract

There is ample empirical evidence that the marginal propensity to consume (MPC) out of transitory income decreases with the level of disposable income. I propose a New Keynesian DSGE model featuring heterogeneous households that allows matching empirical estimates of the MPC distribution. I find that a redistribution of 1% of steady-state output from the top half to the bottom half increases output by 0.14% relative to its steady-state level in the short run. The size of the redistribution multiplier almost doubles when only the top 10% and bottom 10% are targeted, going up to 0.28% of steady-state output.

JEL classification: E21, E61, E62.
Keywords: New-Keynesian models, household heterogeneity, redistributive policy, fiscal stimulus.

2.1 Introduction

In many countries, tax-and-transfer reforms have been implemented to stimulate the economy in the aftermath of the Great Recession. Oh and Reis (2012) estimate that 75% of the increase in US government spending between 2007 and 2009 are transfers (with similar proportions in other OECD countries). A special type of transfers - a budget-neutral combination of negative and positive transfers targeted at different segments of the population - can potentially raise aggregate demand without requiring debt-financing, which is especially relevant when fiscal space is limited. The basis for such policies lies in the expectation that increasing the available income of poor households will increase aggregate demand to a greater extent than the reduction of income of richer households will dampen it. This expectation is supported by a wealth of empirical evidence documenting that the marginal propensity to consume (MPC) out of transitory income decreases in the level of disposable income. This paper adds to recent efforts[1] to incorporate household heterogeneity into a New Keynesian (NK) model by proposing a simple modelling strategy that allows matching empirical estimates of the MPC distribution.

It is well established in the empirical literature that the average MPC out of transitory income is considerably greater than zero.[2] More recent evidence adds to this finding by documenting a negative relationship between the MPC and wealth. For instance, Jappelli and Pistaferri (2014) use Italian survey data and find that the MPC decreases from 0.65 for the poorest percentile to 0.35 for the richest percentile (measured in terms of cash on hand). Their finding is supported by a wealth of empirical papers using micro data from natural experiments or semi-structural methodologies.[3]

Intuitively, giving an extra dollar to the rich will have a different effect from giving it to the poor. A poor, possibly liquidity-constrained household is likely to spend it on much needed consumption goods. Rich, consumption-smoothing households, on the other hand, might save the extra dollar instead of spending it immediately. This consideration has direct consequences for the effectiveness of fiscal policy, as a transitory increase in households' income only affects aggregate demand through the increase in consumption of households who do not (perfectly) smooth consumption. Redistributive policies have not been in the focus of macroeconomists for a long time. One potential explanation is that effective redistribute policies are ruled out in standard representative-agent models. In traditional Dynamic Stochastic General Equilibrium (DSGE) models, the representative consumer behaves like a permanent-income consumer. That means that she consumes a constant fraction of her life-time income, which is only marginally affected by a one-off transfer, so her MPC out of transitory income is close to zero.

This paper suggests a simple approach to introduce an empirically plausible distribution of the MPC across households in NK DSGE models, which allows a meaningful analysis of the role of household

[1] See, e.g., Giambattista and Pennings (2017) and Oh and Reis (2012).

[2] Seminal papers by Campbell and Mankiw (1989) and Deaton (1992) use time-series evidence to cast doubt on the previously widely accepted permanent income hypothesis by documenting a strong dependency of consumption on current income. Another strand of literature uses micro data from natural experiments such as unexpected tax cuts or tax refunds. These studies typically find an average MPC between 0.2 and 0.6. See Jappelli and Pistaferri (2010) for an overview.

[3] For a detailed discussion of the different approaches, see subsection 2.2.4.

heterogeneity for the effectiveness of redistributive fiscal policy. A distribution of MPC is introduced by partitioning the household population into different segments with a varying share of credit-constrained consumers. In this set-up, I analyze the response of the economy to a budget-neutral redistribution, and ask what type of redistribution gives "the most bang for the buck," which is especially relevant for countries with limited fiscal space.[4] There are two important advantages relative to two-agent NK (TANK) models. First, in a TANK model it is only possible to analyze redistributive policies, which are perfectly targeted, i.e., taxing all Ricardian households and giving transfers to all RoT households. This degree of targeting is clearly unrealistic and overstates the effectiveness of government transfers. In the present set-up, lump-sum taxes and transfers can only be targeted to subgroups of the MPC distribution, which yields quantitatively more realistic multipliers. Second, in this model, one can compare the effectiveness of different degrees of targeting in terms of affected size of the population.

This approach is ad-hoc and lacks a thorough micro-foundation of the dependency of the MPC on income or wealth. Yet, it provides a valuable approximation and there is no obvious modelling alternative to generate quantitatively meaningful transfer multipliers. This is because direct consumption effects are arguably the crucial feature for a quantitatively meaningful assessment of redistributive policies, i.e., the model's ability to be calibrated to match empirical estimates of the MPC distribution is key.[5] The endogenously generated MPC distribution obtained by standard incomplete-market models fails to generate the substantial MPC heterogeneity found in the data.[6] Moreover, the potential drawback that the ad-hoc MPC distribution in the model at hand is constant is most likely to be small, as changes in the MPC due to a redistribution are negligibly small. Chetty and Finkelstein (2013) argue that consumption (and not wealth or income) heterogeneity is most relevant for analyzing both social welfare and redistributive policies. A further advantage of the approach suggested in the paper at hand is that it can be easily incorporated into large-scale DSGE models, which allows to study the role of MPC inequality for a large variety of research questions.

In the baseline calibration, I find that a transitory redistribution of 1% of steady-state output from the top half to the bottom half of the MPC distribution increases output on impact by 0.14%.[7] The size of the impact multiplier considerably increases with the degree of targeting of the redistribution, i.e., the more it is targeted to the extremes of the distribution. It almost doubles when only smaller groups at the very top and the very bottom are targeted, going up to 0.28 for a redistribution from the top 10% to the bottom 10%. For a given degree of targeting, a redistribution is more expansive the larger the paying group relative to the receiving group.

Let me relate to the literature on the aggregate effects of redistributive policies. Jappelli and Pistaferri (2014) use their estimation of the distribution of MPC to evaluate the effects of a budget-neutral

[4] There is a consensus that multipliers of debt-financed government purchases are generally larger, at least in normal times (see, e.g., Giambattista and Pennings (2017) and McKay and Reis, 2016). However, for fiscally-constrained countries it is often undesirable to take on more debt.

[5] Acharya and Dogra (2020) argue that it is not MPC heterogeneity, but mainly uninsurable risk and resulting precautionary savings that determine the fiscal purchases multiplier. However, their results rely on the assumption of the cyclicality of income risk and do not feature multipliers of (targeted) transfers.

[6] See also the literature review later in this section.

[7] Top (bottom) refers to the part with a lower (higher) MPC.

redistribution of 1% of national disposable income in a partial-equilibrium framework. They find that a redistribution from the top 10% to the bottom 10% generates an increase of aggregate consumption of 0.35%.[8] The main difference to my general-equilibrium analysis is that they only take into account the direct consumption responses, but ignore feedback effects stemming, e.g., from the adjustment of labor supply or wages, which dampens the effects on aggregate demand in my set-up relative to theirs.

There is a growing strand of literature that follows Galí et al. (2007) in including Rule-of-Thumb (RoT) consumers into an otherwise standard NK model to induce a positive dependency of consumption on current income. These papers typically argue that two types of heterogeneous households are sufficient to capture the most important implications of heterogeneity for fiscal policy. It is implicitly assumed that it is mainly the difference in consumption responses between constrained and unconstrained agents driving the response of aggregate consumption, abstracting from the heterogeneity within the group of unconstrained households. In the context of monetary policy, this is argued by Debortoli and Galí (2017) who find that their ad-hoc model can approximate the responses of a simple incomplete-market model both qualitatively and quantitatively. However, as noted by the authors themselves, "*[n]eedless to say, TANK models might not constitute a good approximation for richer HANK models, or more importantly the actual data.*" The degree of MPC heterogeneity found in the data can, however, not be reproduced by the simple HANK model that they use as a benchmark, and not even by richer HANK models. Giambattista and Pennings (2017) compute multipliers of government purchases and transfers in a medium-scale TANK model. Their set-up is nested as a special case in the model outlined in this paper when we choose the limiting case of only two segments (with a MPC of one and zero) instead of introducing a fully-fledged MPC distribution. A further key difference to their paper pertains to the nature of the experiment: They analyze a targeted transfer that is financed by a lump-sum tax on all Ricardian households and that is paid out to all RoT consumers, while I do not assume that the transfer can be perfectly targeted. They find a multiplier of 0.25 for a redistribution of 1% of output and for a share of 30% of RoT consumers, which is significantly higher than my value. As Giambattista and Pennings (2017) note, "*[i]n the real world, it is unlikely that a government could perfectly target transfers [...], and so the targeted transfer multipliers [...] are an upper bound.*" Therefore, they also report multipliers when targeting is imperfect, i.e., when also RoT consumers pay lump-sum taxes and/or Ricardian households receive lump-sum transfers. In this case, the multiplier is considerably smaller, but still larger than mine, which is likely to be due to the difference in modelling the heterogeneity in MPC.

A different path to modeling household heterogeneity is taken by the literature combining the incom- plete-market framework with price rigidities and aggregate shocks. This class of models builds on the classic Bewley (1977) model where a wealth distribution endogenously results as consequence of agents' precautionary savings to buffer against idiosyncratic shocks. At the same time, a distribution of MPC is generated endogenously, as households with different wealth levels react differently to changes in their current income.[9] Oh and Reis (2012) use an incomplete-market model, where households face idiosyncratic shocks to their health and to their employment status, to study the effectiveness of redis-

[8] See figure 5 in Jappelli and Pistaferri (2014).

[9] The implications of household heterogeneity for monetary policy are, e.g., analyzed by Auclert (2019), McKay et al. (2016), Kaplan et al. (2018), Farhi and Werning (2019), Gornemann et al. (2016), and Luetticke (forthcoming).

tributive policies.[10] The advantage here is that the MPC of a household changes whenever its position in the health-employment distribution changes, while this is not the case for the ad-hoc distribution used in this paper. However, a potential drawback is that with an empirically supported calibration of the idiosyncratic shock process, it is challenging to generate an MPC distribution that matches the data in terms of inequality. Oh and Reis (2012) analyze a redistribution of 3.4% GDP (distributed over two years) along the health-employment distribution, where they evaluate the effects of the split that achieves the biggest expansion. They find a peak percentage increase of consumption of around 0.07. They attribute the low value of the multiplier partially to an unrealistically low average MPC (11%), which mutes the "Keynesian" effect in their model.

The paper is organized as follows. In Section 2.2, I present the model and the calibration. Section 2.3 explores the transmission mechanism and computes the multipliers associated with different redistributive policies. Section 2.4 concludes.

2.2 Model

I propose a model which allows to isolate the role of household heterogeneity. A discrete distribution of MPC is introduced by partitioning the household population into different segments with a varying ratio of Ricardian versus RoT consumers.

2.2.1 Firms

The model features three types of firms, two of which are standard in the literature. The only non-standard feature is a labor-bundler firm for each segment, which is competitive and bundles the labor of all workers in that segment in a cost-minimizing way. Labor bundles from all N segments are used as input by monopolistic intermediate-good firms. A competitive final-good firm bundles good varieties into the final good. There are infinitely many intermediate-good producers $i \epsilon [0, 1]$, where each firm produces one differentiated good variety and has market power. Prices can only be set in a staggered fashion.

Labor-bundler firms

There is one firm in each labor market segment $n \epsilon \{1, ..., N\}$ that bundles labor from that segment into a labor bundle. The index n denotes both the segment and the respective labor-bundler firm. There is a unity mass of households in total; each household can only work in one production segment. Labor-bundler firms are competitive, so the labor bundler of segment n chooses the demand for type-j labor, $h_t^n(j)$, that minimizes the aggregate wage bill of that segment such that the aggregation over individual labor types into the segment-n labor aggregate, H_t^n, is cost-efficient

$$\min_{h_t^n(j)} \quad \int_{z_{n-1}}^{z_n} w_t^n(j) h_t^n(j) dj$$

[10] Other papers that analyze the implications of household heterogeneity for fiscal policy include Bilbiie et al. (2013), Oh and Reis (2012), McKay and Reis (2016), and Hagedorn et al. (2019).

$$\text{s.t. } H_t^n = \left[\gamma_n^{-\frac{1}{\varepsilon_n}} \int_{z_{n-1}}^{z_n} h_t^n(j)^{\frac{\epsilon_n - 1}{\epsilon_n}} \, dj \right]^{\frac{\varepsilon_n}{\varepsilon_{n-1}}},$$

where $w_t^n(j)$ is the nominal wage of worker j from segment n, $h_t^n(j)$ are the hours of worker j from segment n, γ_n is the size of segment n, and $[z_{n-1}, z_n]$ is the sub-interval of households belonging to segment n. Parameter $\varepsilon_n > 1$ is the elasticity of substitution between types of labor in one production segment, which determines workers' market power in setting their wages. Cost minimization yields the demand for type-j labor in segment n

$$h_t^n(j) = \frac{1}{\gamma_n} \left(\frac{w_t^n(j)}{W_t^n} \right)^{-\varepsilon_n} H_t^n. \tag{2.1}$$

The demand for worker j from segment n depends on the average demand for such a worker, $\frac{H_t^n}{\gamma_n}$, and the worker's wage relative to wage index in segment n, $\frac{w_t^n(j)}{W_t^n}$. The wage index in segment n, W_t^n, which is the price for one unit of labor aggregate, H_t^n, is therefore given by

$$W_t^n = \left[\frac{1}{\gamma_n} \int_{z_{n-1}}^{z_n} \left(w_t^n(j) \right)^{1-\varepsilon_n} dj \right]^{\frac{1}{1-\varepsilon_n}}. \tag{2.2}$$

Intermediate-good firms

Each intermediate-good firm i produces a good variety labelled i. Good varieties are imperfect substitutes, which makes intermediate-good firms compete monopolistically, and prices are set in a staggered fashion. Production of good i requires $Y_t(i) = A_t N_t(i)$, where hours employed by firm i, $N_t(i)$, are an aggregate of the N labor bundles. Firm i minimizes the expenditures for inputs $H_t^1(i), \ldots, H_t^N(i)$ in a cost-minimizing way

$$\min_{H_t^n(i)} W_t^1 H_t^1(i) + \ldots + W_t^N H_t^N(i)$$

$$\text{s.t. } N_t(i) = \left[\omega_1^{\frac{1}{\varepsilon_m}} \left(H_t^1(i) \right)^{\frac{\varepsilon_m - 1}{\varepsilon_m}} + \ldots + \omega_N^{\frac{1}{\varepsilon_m}} \left(H_t^N(i) \right)^{\frac{\varepsilon_m - 1}{\varepsilon_m}} \right]^{\frac{\epsilon_m}{\epsilon_m - 1}},$$

where $\varepsilon_m > 1$ is the elasticity of substitution between different labor bundles and ω_n is segment n's production share. If $\gamma_n = \omega_n$, segment n's size is equivalent to its share in production. The optimization yields the demand for bundle n from firm i

$$H_t^n(i) = \omega_n \left(\frac{W_t^n}{W_t} \right)^{-\varepsilon_m} N_t(i) \tag{2.3}$$

and the economy-wide wage index

$$W_t = \left[\omega_1 \left(W_t^1 \right)^{1-\varepsilon_m} + \omega_2 \left(W_t^2 \right)^{1-\varepsilon_m} + \ldots + \omega_N \left(W_t^N \right)^{1-\varepsilon_m} \right]^{\frac{1}{1-\varepsilon_m}}. \tag{2.4}$$

Aggregation of labor demand schedules

Since I assume that intermediate-good firms are homogeneous and have the same production technology, all firms demand the same relative amount of each labor bundle n. This allows me to aggregate the demand schedules for labor bundle n from the different intermediate-good firms as $H_t^n = \int_0^1 H_t^n(i)\,di$, so from (2.3) it follows that

$$H_t^n = \omega_n \left(\frac{W_t^n}{W_t}\right)^{-\varepsilon_m} \int_0^1 N_t(i)\,di = \omega_n \left(\frac{W_t^n}{W_t}\right)^{-\varepsilon_m} N_t, \tag{2.5}$$

where $N_t \equiv \int_0^1 N_t(i)$ is aggregate labor demand. This implies that total demand for labor bundle n is a function of average demand for segment n, $\omega_n N_t$, and segment n's wage relative to the aggregate wage level, $\frac{W_t^n}{W_t}$. Inserting segment demand (2.5) into the individual demand (2.1) gives us the demand for the labor of worker j in segment n as function of the total labor demand

$$h_t^n(j) = \frac{\omega_n}{\gamma_n} \left(\frac{w_t^n(j)}{W_t^n}\right)^{-\varepsilon_n} \left(\frac{W_t^n}{W_t}\right)^{-\varepsilon_m} N_t. \tag{2.6}$$

Thus, individual demand of worker j is a function of average demand per worker, $\frac{\omega_n}{\gamma_n} N_t$, the ratio of individual wage to segment wage, $\frac{w_t^n(j)}{W_t^n}$, and the ratio of segment wage to aggregate wage, $\frac{W_t^n}{W_t}$.

Price setting

The period profit function of intermediate-good firm i reads

$$\Pi_t(i) = P_t(i)Y_t(i) - W_t^1 H_t^1(i) - W_t^2 H_t^2(i) - ... - W H_t^N(i), \tag{2.7}$$

which denotes nominal revenues minus the costs for labor bundles from different segments. It can be shown that $W_t^1 H_t^1(i) + W_t^2 H_t^2(i) + ... + W_t^N H_t^N(i) = W_t N_t(i)$,[11] so the period profit function simplifies to

$$\Pi_t(i) = P_t(i)Y_t(i) - W_t N_t(i). \tag{2.8}$$

Price rigidities are introduced in the form of staggered price setting à la Calvo (1983). In this framework, each firm is allowed to change its price with probability $(1-\theta)$, which is independent of the time elapsed since the last adjustment. With Calvo pricing, firm i seeks to maximize the discounted value of expected profits, taking into account the possibility that today's newly set price $P_t(i)$ is still valid in future periods. Optimization is constrained by the demand schedule for good i and the production technology.

$$\max_{P_t(i)} \sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} \Pi_{t+k|t} \right\},$$

$$\text{s.t. } Y_{t+k} = A_{t+k} N_{t+k}$$

$$\text{and } Y_{t+k|t} = \left(\frac{P_t(i)}{P_{t+k}}\right)^{-\varepsilon} Y_{t+k},$$

[11] See appendix A.1.2.

where $Q_{t,t+k} \equiv \beta \left\{ \left(\frac{P_t}{P_{t+k}} \right) \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \right\}$ is the stochastic discount factor,[12] $Y_{t+k|t}$ and $\Pi_{t+k|t}$ denote output and profit of a firm in period $t+k$ that has set its price in period t, and ε denotes the price elasticity of demand for variety i.

The first-order necessary condition related to this problem is[13]

$$\sum_{k=0}^{\infty} E_t \left\{ \theta^k Q_{t,t+k} Y_{t+k|t} \left[P_t^* - \frac{\varepsilon}{\varepsilon - 1} MC_{t+k} \right] \right\} = 0, \tag{2.9}$$

where P_t^* is the optimal price in period t and nominal marginal costs in period t are given by $MC_t = \frac{W_t}{A_t}$. The aggregate price index can be represented by a weighted average of the optimally newly set price and last period's price index

$$P_t = \left[\theta (P_{t-1})^{1-\varepsilon} + (1-\theta) \left(P_t^* \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} . \tag{2.10}$$

Final-good firms

Final-good firms are modelled as standard in the literature. They are competitive and bundle good varieties into a final consumption good

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} . \tag{2.11}$$

2.2.2 Households

In each production segment n, there is a different share of Ricardian, consumption-smoothing households, $1-\lambda_n$, and a share of RoT consumers, λ_n. Note that households cannot switch between segments.

Consumption decision

A Ricardian household from segment n maximizes its discounted value of expected life-time utility[14]

$$\max_{\left\{c_{t+k}^{n,ric}, n_{t+k}^{n,ric}\right\}} E_t \left\{ \sum_{k=0}^{\infty} \beta^k \left[\frac{\left(c_{t+k}^{n,ric} \right)^{1-\sigma}}{1-\sigma} - \frac{\left(h_{t+k}^{n,ric} \right)^{1+\phi}}{1+\phi} \right] \right\},$$

where $c_t^{n,ric} \equiv \left[\int_0^1 c_t^{n,ric}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$ is the individual consumption bundle of household j in segment n, which is a composite of the different good varieties. Since all households have the same preferences, the composition of the bundle will be the same for all segments n and for both Ricardian and RoT

[12] This stochastic discount factor is used because intermediate-good firms are owned by Ricardian households. See, e.g., Galí et al. (2007).

[13] See appendix A.1.3.

[14] I suppress subscript j, as each Ricardian household in that segment faces the same problem.

consumers, so $c_t \equiv \left[\int_0^1 c_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$. The period budget constraint of the Ricardian household is given by

$$P_t c_t^{n,ric} + E_t\left[Q_{t,t+1}\, b_{t+1}^n\right] = b_t^n + w_t^n h_t^n + \pi_t^n + t_t^n,$$

where b_t^n denotes bond holdings of this consumer working in segment n, π_t^n denotes her profit income, and t_t^n denotes her lump-sum taxes or transfers.

There is one associated Euler equation for each segment n

$$\frac{1}{R_t} = \beta E_t \left\{ \left[\frac{c_{t+1}^{n,ric}}{c_t^{n,ric}}\right]^{-\sigma} \frac{P_t}{P_{t+1}} \right\}. \tag{2.12}$$

As households cannot switch between segments, there is a representative Ricardian consumer in each segment and a corresponding Euler equation. I face a technical problem with respect to the model's stationarity,[15] as there is the possibility that consumption levels of segments permanently diverge and not return to their steady state, following a shock. That is because there are multiple possible combinations of segment consumption adjustments to result in the same adjustment of aggregate consumption, which is pinned down in equilibrium. For instance, in response to a transfer shock, consumption of a transfer-paying segment decreases, while consumption of a transfer-receiving segment increases and there is no mechanism in the model to ensure that these revert back to their steady-state levels.

I therefore introduce a term that ensures that $c_t^{n,ric}$ returns to the steady-state average Ricardian consumption $\overline{c}^{ric}$ in the long run.[16]

$$\left(R_t + \psi\left[c_t^{n,ric} - \overline{c}^{ric}\right]\right)^{-1} = \beta E_t \left\{ \left[\frac{c_{t+1}^{n,ric}}{c_t^{n,ric}}\right]^{-\sigma} \frac{P_t}{P_{t+1}} \right\}. \tag{2.13}$$

This technical fix ensures that the dispersion in segment consumption levels vanishes asymptotically. Note that this has no implications for the short-term and medium-term dynamics, as parameter ψ is chosen to be 0.0001.

RoT consumers do not have access to financial markets and do not own firms. Assuming that they work the same hours as Ricardian households, as standard in the literature,[17] their budget constraint is given by

$$P_t c_t^{rot,n} = w_t^n h_t^n + t_t^n, \tag{2.14}$$

which implies that RoT households consume all of their income in every period.

Consumption in one labor-market segment n is then given by the weighted average consumption of Ricardian and RoT households in that segment

$$c_t^n = \lambda_n\, c_t^{rot,n} + (1-\lambda_n)\, c_t^{ric,n}. \tag{2.15}$$

[15]The model is solved by perturbation methods, i.e., the model is linearized around its steady state. A necessary condition for this method to work is thus the existence of a unique steady state, which is not given for a non-stationary model.

[16]This modeling choice is based on Schmitt-Grohé and Uribe (2003)'s idea to render a small-open-economy model stationary. They introduce a risk premium for the interest rate parity condition that forces external debt in the long term to return to the initial level.

[17]See, e.g., Drautzburg and Uhlig (2015).

Wage setting

As each household j supplies a differentiated type of labor, it exerts market power and sets the wage that maximizes its expected lifetime utility. Following Erceg et al. (2000), I assume that wages are set in a staggered fashion. The wage can be re-adjusted with a probability $1-\theta_w$ in each period, irrespective of the time of the last wage adjustment. I assume that Ricardian and non-Ricardian supply the same type of labor, work the same number of hours, and earn the same wage. However, consumption and hence their marginal utility of consumption differs. In the following I assume that the labor union takes into account a weighted average of the marginal utility of consumption, $\left(c^n_{t+k}\right)^{-\sigma} = \left(\lambda_n\, c_t^{n,ric} + (1-\lambda_n)\, c_t^{n,rot}\right)^{-\sigma}$.[18] The optimization problem of labor union j from segment n looks as follows

$$\max_{w_t^n} E_t \left\{ \sum_{k=0}^{\infty} (\beta\theta_w)^k \left[\frac{\left(w_t^n h^n_{t+k|t}/P_{t+k}\right)^{1-\sigma}}{1-\sigma} - \frac{\left(h^n_{t+k|t}\right)^{1+\phi}}{1+\phi} \right] \right\},$$

taking into account the labor demand for labor from segment n given by equation (2.6), $h_{\,t+k|t}$.

The associated first-order condition is[19]

$$E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k\, N_{t+k} \left(W^{n,r}_{t+k}\right)^{\varepsilon_n - \varepsilon_m} \left(W^r_{t+k}\right)^{\varepsilon_m} \left[\left(c^n_{t+k}\right)^{-\sigma} \left(\frac{w_t^{n*}}{P_{t+k}}\right) - \frac{\varepsilon_n}{\varepsilon_n - 1} \left(h^n_{t+k}\right)^{\phi} \right] = 0, \quad (2.16)$$

where w_t^{n*} is the optimal wage set by union n that resets the wage in period t, $W_t^{n,r} \equiv \frac{W_t^n}{P_t}$ is the real wage in sector n, and $W_t^r \equiv \frac{W_t}{P_t}$ is the real aggregate wage index, which is relevant for firms' marginal costs. In optimum, the wage is chosen such that the real wage is, on average, equal to a mark-up over the marginal rate of substitution between consumption and labor.

Analogously to the price setting problem, the wage index of sector n evolves according to

$$W_t^n = \left[\theta_w \left(W^n_{t-1}\right)^{1-\varepsilon_m} + (1-\theta_w) \left(W_t^{n,*}\right)^{1-\varepsilon_m}\right]^{\frac{1}{1-\varepsilon_m}} . \quad (2.17)$$

As the probability of being able to adjust the wage is the same in every sector, the aggregate wage index evolves in a similar way

$$W_t = \left[\theta_w \left(W_{t-1}\right)^{1-\varepsilon_m} + (1-\theta_w) \left(W_t^{*}\right)^{1-\varepsilon_m}\right]^{\frac{1}{1-\varepsilon_m}} , \quad (2.18)$$

where

$$W_t^* \equiv \left[\omega_1 \left(W_t^{1,*}\right)^{1-\varepsilon_m} + \omega_2 \left(W_t^{2,*}\right)^{1-\varepsilon_m} + \ldots + \omega_N \left(W_t^{N,*}\right)^{1-\varepsilon_m}\right]^{\frac{1}{1-\varepsilon_m}} . \quad (2.19)$$

Aggregate employment

Using equation (2.5) and the production function, market clearing for employment in segment n implies[20]

$$H_t^n = s_t^P \left(\frac{W_t^n}{W_t}\right)^{-\varepsilon_m} \omega_n \frac{Y_t}{A_t}, \quad (2.20)$$

[18] Drautzburg and Uhlig (2015) use a similar approach.
[19] For the derivation, see appendix A.1.4.
[20] For the derivation, see appendix A.1.5.

where $s_t^P \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} di$ denotes price dispersion. Due to inefficiencies caused by price dispersion and by the difference of segment-n wage from average wage, the production of the share of output to be covered by segment n, $\omega_n \frac{Y_t}{A_t}$, requires more than one unit of labor bundle H_t^n.[21] Substituting equation (2.20) into the individual demand (2.1), I obtain

$$h_t^n(j) = \frac{\omega_n}{\gamma_n} \left(\frac{w_t^n}{W_t^n}\right)^{-\varepsilon_n} \left(\frac{W_t^n}{W_t}\right)^{-\varepsilon_m} s_t^P \frac{Y_t}{A_t}. \tag{2.21}$$

Integrating over all workers j, it can be shown that[22]

$$N_t = s_t^P \frac{Y_t}{A_t} \left[g_t^1 s_t^{w,1} + ... + g_t^N s_t^{w,N}\right], \tag{2.22}$$

where $s_t^{w,n} \equiv \frac{1}{\gamma_n} \int_{z_{n-1}}^{z_n} \left(\frac{w_t^n}{W_t^n}\right)^{-\varepsilon_n} dj$ denotes the wage dispersion within one sector and $g_t^n \equiv \omega_n \left(\frac{W_t^n}{W_t}\right)^{-\varepsilon_m}$ is a weight that accounts for the difference of segment-n wage and average wage that reflects the demand of the intermediate firm for segment-n labor. Total wage dispersion is defined as a weighted average of intra-segment wage dispersion $s_t^W \equiv \left[g_t^1 s_t^{w,1} + ... + g_t^N s_t^{w,N}\right]$, so market clearing of aggregate employment requires

$$N_t = s_t^P s_t^W \frac{Y_t}{A_t}. \tag{2.23}$$

Equation (2.23) implies that the production of one unit of output requires more than one unit of aggregate employment due to inefficiencies caused by price and wage dispersion.

2.2.3 Government Sector and Monetary Policy

As the focus is on transfer policies, I assume that the government does not make purchases and conducts a budget-neutral redistribution by taking an amount T_t from one subset of the household population and transferring it to another subset of the population. The transfer shock evolves according to the following AR(1) process

$$T_t = \rho_T \, T_{t-1} + \varepsilon_t, \tag{2.24}$$

with $0 < \rho_T < 1$ and transfer shock ε_t is i.i.d. with zero mean and standard error 0.009, which implies a redistribution of 1% of steady-state output. Let N_{top} denote the number of segments belonging to the subset of households paying out transfers and N_{bottom} the number of segments belonging to the subset of segments the transfer will be paid out to. I assume that the redistribution always happens from the top to the bottom of the MPC distribution and that transfer payments (receipts) are evenly distributed across the households of the respective sub-sample. The share of the transfer to be paid/received by segment n corresponds to the population share of segment n. Accordingly, the shock for a transfer-paying segment, $n \, \epsilon \, \{1, ..., N_{top}\}$, will be

$$T_t^n = -T * \frac{\gamma_n}{\tau_1 + ... + \gamma_{N_{top}}}, \tag{2.25}$$

[21] $s_t^P < 1$.

[22] For the derivation, see appendix A.1.5.

while the shock for a transfer-receiving segment $n \epsilon \{N - N_{bottom} + 1, ..., N\}$ is

$$T_t^n = T * \frac{\gamma_n}{\gamma_{N-N_{bottom}+1} + ... + \gamma_N}. \tag{2.26}$$

The direct change in per-capita current income in a transfer-paying segment is

$$t_t^n = -\frac{T}{\gamma_1 + ... + \gamma_{N_{top}}}, \tag{2.27}$$

and the analogue for a transfer-receiving segment is

$$t_t^n = \frac{T}{\gamma_{N-N_{bottom}+1} + ... + \gamma_N}. \tag{2.28}$$

Note that if not all segments are targeted by the transfer (e.g., if we redistribute from the top 10% to the bottom 10%), the segments not directly affected will have a transfer of 0.

Monetary policy follows a standard Taylor Rule targeting inflation

$$R_t = \frac{1}{\beta}\left(\frac{P_t}{P_{t-1}}\right)^{\phi}. \tag{2.29}$$

2.2.4 Calibration

The calibration of the distribution of MPC according to empirical evidence is crucial for my experiment. There are three different empirical strategies to estimate this distribution, each of which has advantages and drawbacks, but lead in their combination to a robust impression of the negative relationship between the MPC and some sort of liquid wealth.

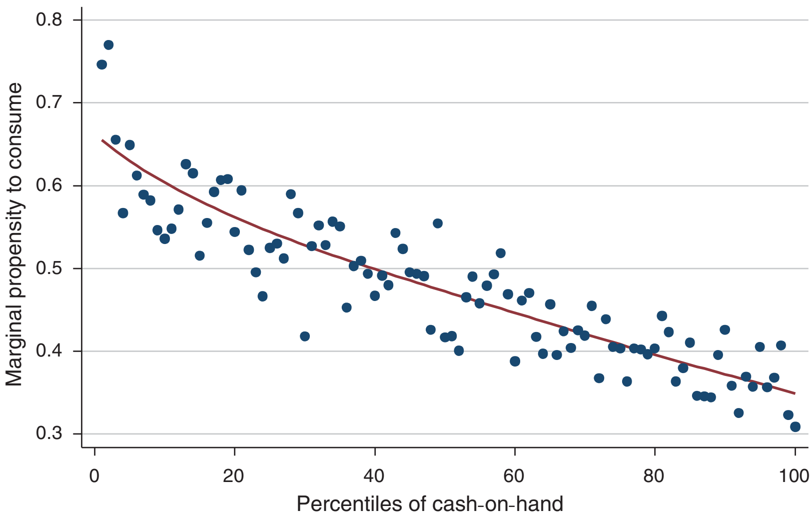
The first approach uses micro data from natural experiments to estimate the MPC controlling for different levels of some form of liquid funds. As this data is usually only available for broad brackets, a drawback of this approach is that it is not possible to obtain a fine empirical distribution of MPC over wealth. For the 2001 US income tax rebates, Dynan et al. (2004) find a positive relationship between the marginal propensity to save and lifetime income, which is supported by Johnson et al. (2006) who find a stronger consumption response of households with low liquid wealth and low income.

A different route is taken by Kaplan et al. (2014) who employ a semi-structural method to back out MPC of transitory income shocks using panel data. This method is subject to a similar drawback as the first approach, as the analysis can only be conducted at the group level, and not for individual households. Kaplan et al. (2014) find an overall negative correlation of wealth and MPC, with some exceptions at the very top of the wealth distribution, potentially pointing to the existence of some "wealthy hand-to-mouth consumers" (see Kaplan et al., 2014).[23]

A third approach is to use data from a survey asking how much of an unexpected hypothetical tax refund the household would spend. This set-up enables the computation of the MPC at the level of a single household. It also rules out endogeneity problems, because households are only asked about a

[23]Kaplan et al. (2014) use this term to describe households with high levels of illiquid wealth, but low levels of liquid wealth, which makes them strongly respond to changes in current income.

Figure 2.1: **Average MPC by cash-on-hand percentiles.**



Source: Jappelli and Pistaferri (2014).

Note: Cash on hand is defined as the sum of household disposable income and financial wealth, net of consumer debt.

hypothetical change in income. Jappelli and Pistaferri (2014) analyze such data from the 2010 Italian Survey of Household Income and Wealth to compute MPC for each percentile of cash-on-hand, defined as the sum of household disposable income and financial wealth, net of consumer debt. Figure 2.1 shows the resulting negative correlation between cash-on-hand and the average MPC of the respective percentile. Note that the relationship has the form of a hockey stick, i.e., the relationship is approximately linear for deciles 30-100, while for very poor deciles the MPC decreases more strongly.

While this last approach has the advantage to provide an arbitrarily fine grid of wealth bins, there are also potential drawbacks of self-reported MPC, the most important being that the answers might not be informative about how households would behave facing a real change in income. To get a more robust idea of the relationship, Auclert (2019) uses all of the three different approaches used in the literature and three different indicators of wealth. Across methods and measures, he finds a negative relationship, which is however less clear for the top of the distribution when using the first two approaches.[24] As Kaplan et al. (2014) and Auclert (2019) point out, this could hint at an important role of the liquidity of assets for the MPC. I will use Jappelli and Pistaferri (2014)'s estimates as my calibration target in the benchmark case. In a robustness exercise, I will investigate the implications of a more u-shaped distribution.

[24]See figure A.1.1 in appendix A.1.1.

Table 2.1: **Calibration of model parameters.**

Parameter	Value	Source/target
Discount factor	0.99	Annual risk-free rate of 4%
Relative risk aversion	1	Log-utility
Frisch elasticity of labor supply	1	Kimball and Shapiro (2008)
Elasticity of subst. goods varieties	10	11% price markup, Basu and Kimball (1997)
Elasticity of subst. worker types	7.4	15% wage markup, Chari et al. (2002)
Calvo probability firms	0.6875	Avg. lifetime 9.6 months, Druant et al. (2009)
Calvo probability unions	0.76	Avg. lifetime 12.5 months, Druant et al. (2009)
Inflation coefficient in Taylor Rule	1.5	Standard
Persistence of transfer shock	0.9	Target: ARRA 2009 redistribution

2.3 Results

I follow Giambattista and Pennings (2017) in looking at a redistribution of 1% of steady-state output. They argue that, according to the narrowest classification, transfer payments from the 2009 American Recovery and Reinvestment Act (ARRA) can be well approximated by an AR(1) process with persistence 0.9. As they use the most conservative classification of transfers, the results can be seen as a lower bound of the ARRA's impact.[25] As a robustness check, I will also consider a one-off shock that is more in line with the Bush stimulus payments in the years 2001 and 2008. The calibration of the remaining parameters is standard in the literature and is listed in table 2.1. I first employ a model that features only two segments to analyze the transmission mechanism of transfer shock. In a second step, I use the full model with a plausibly calibrated distribution of MPC to compile multipliers for different splits of the redistribution.

2.3.1 The Transmission Mechanism

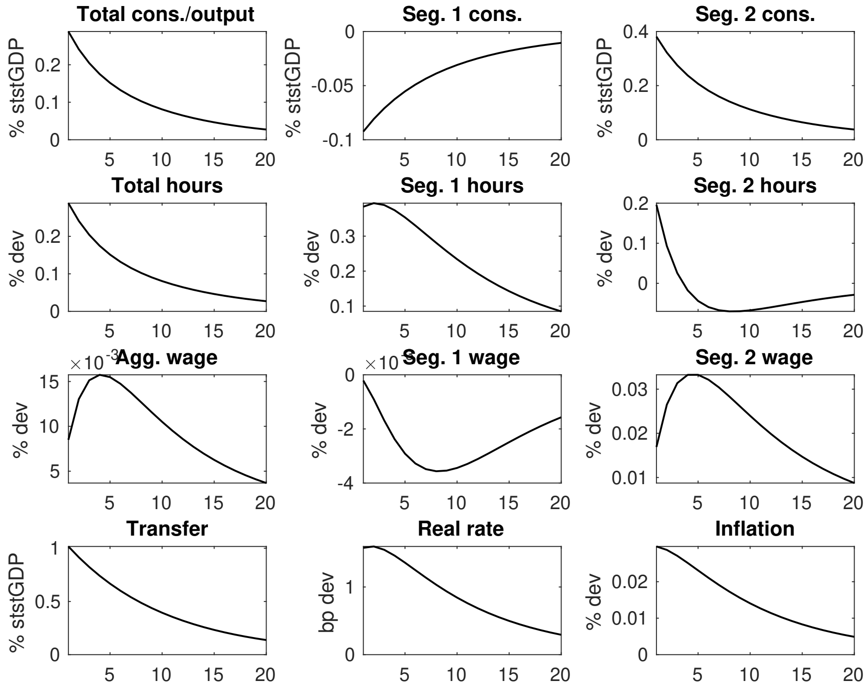
Consider the model featuring only two distinct segments to focus on the qualitative implications of the model and to build intuition for the mechanisms at work. I assume that both segments are symmetric in each aspect other than the share of constrained consumers, i.e., they are of equal size and have the same share in production. The two segments differ in their average MPC, i.e., in their share of RoT consumers. In the following, I assume that segment 1 is the rich segment, which includes only 20% RoT consumers, while segment 2 features 60% RoT consumers. [26] Figure 2.2 shows the impulse response functions (IRF) of both aggregate and segment-specific variables to a redistributive shock, taking 1% of steady-state output from the rich segment and transferring it to the poor segment. Unless stated

[25] See the online appendix of Giambattista and Pennings (2017) for more details on their computations.

[26] As I am only interested in qualitative implications in this section, the exact numbers do not matter as long as the rich segment has a lower MPC than the poor segment. However, I made this choice based on the overview of Jappelli and Pistaferri (2010) who report MPC for the whole population between 0.2 and 0.6.

otherwise, variables are expressed in percent deviations from steady state. The top left panel shows that this redistribution expands aggregate consumption by about 0.3% of steady-state output (mirrored by an equal increase in output, as the model does not feature investment or government purchases).

Figure 2.2: **IRF to a transfer shock in the TANK model with rigid wages.**



Note: Here, the transfer shock redistributes 1% of st.st. output from segment 1 to segment 2.

In order to understand this expansion, consider first the direct effects of the redistribution, i.e., the change of segment consumption in response to the change in current income. A segment's consumption response crucially depends on the share of RoT consumers in this segment, as the targeted transfer only directly affects RoT consumption, while the behavior of Ricardian consumers is solely governed by the respective Euler equation. Note that Ricardian consumers will indirectly influence RoT consumption though, as both RoT and Ricardian consumers work the same number of hours. First, consider the effect on the transfer-paying segment 1. RoT consumers in segment 1 decrease their consumption in response to the negative income shock, while Ricardian consumption remains unchanged. With perfectly flexible wages, this negative wealth effect leads both RoT and Ricardian consumers to work more hours, which increases their current income. This almost entirely neutralizes the negative effect from the transfer payment on consumption. With rigid wages, however, the adjustment happens via a fall in the wage mark-up.[27] The mechanics work in the opposite direction in the transfer-receiving sector.

[27] As workers have market power, hours worked are completely determined by firms' labor demand. When wages adjust in a

With rigid wages preventing the perfect adjustment of the labor supply, there is an increase in aggregate consumption, driven by an increase in consumption of the transfer-receiving segment, which is much stronger than the decrease of consumption in the transfer-paying segment.[28] The asymmetric adjustment is caused by the heterogeneous levels of MPC in the two segments: The transfer-receiving segment has a higher MPC (i.e., share of RoT consumers), so it increases its consumption in response to the transfer shock more strongly than the other segment decreases its consumption.

The hike in output symmetrically increases demand for labor from both segments, as the segments have equal production shares. The increased demand for labor strengthens the increase in hours in segment 1 (due to the wealth effect on labor supply), while it offsets the decrease in hours in segment 2. The wage in segment 2 increases as result of the increased marginal rate of substitution (MRS) between consumption and labor (driven by the strong upward adjustment of consumption in this segment), while the wage in segment 1 decreases mildly (caused by the decrease in consumption and the increase in hours). This leads to a very mild wage and, via the increase in marginal costs, to price inflation. In response, the central bank increases the nominal interest rate.

These considerations show that the expansionary effect of a redistribution, which is not perfectly targeted from Ricardian to RoT consumers, crucially depends on the presence of wage stickiness, as the degree of wage stickiness determines the labor-supply response. With perfectly flexible wages, hours worked would be chosen in a way to fully stabilize current income, i.e., to fully finance the decrease in the transfer-paying sector and to fully neutralize the increase in the transfer-receiving sector (see figure 2.3). There is a strong consensus in the empirical literature that wage rigidity is a quantitatively important friction in labor-market adjustment.[29]

2.3.2 Multipliers

In order to derive quantitatively meaningful multipliers, I employ a model with ten different segments and calibrate the shares to match the empirically supported MPC distribution.[30] Multipliers are computed for different splits of the same budget-neutral redistribution, i.e., for varying sizes of the transfer-paying subset and different sizes of the transfer-receiving group. Being able to compare multipliers for different splits to see which redistribution split gives the most "bang for the bug" is an advantage of my approach relative to TANK models. In particular, I can also study types of redistributions, which do not affect an arbitrary group in the middle of the distribution, e.g., a redistribution from the top 10 to the bottom 20%. Table 2.2 shows different multipliers for different splits of the redistribution.[31] I compute the impact multiplier, the average multiplier after 5, 10, and 20 years, as well as the net-present-value (NPV) multiplier after 5, 10, and 20 years (all defined in percent of steady state GDP). Giambattista and Pennings (2017) compute the NPV multiplier as defined in Uhlig (2010), i.e., they take the ratio of the present value of GDP changes and the present value of transfer changes. Since the fiscal measures con-

staggered fashion, also hours adjust in this way.

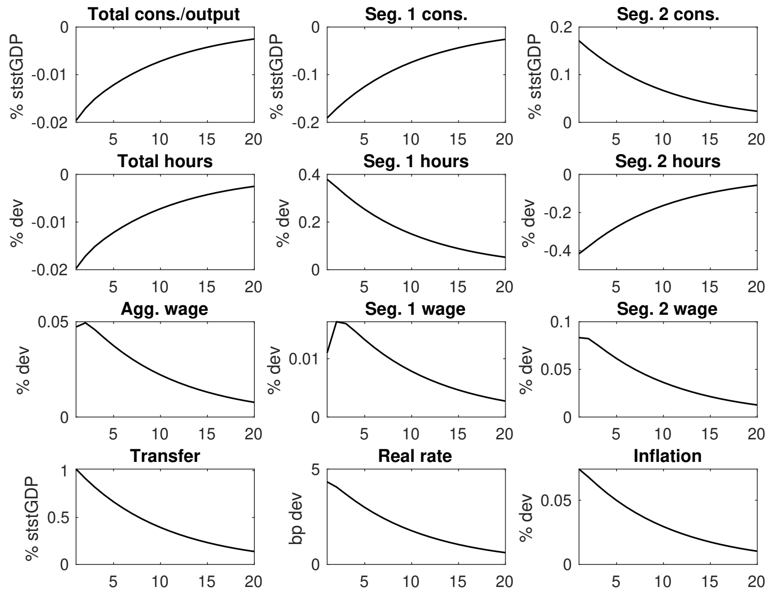
[28]Note that segment-specific consumption is depicted in units of economy-wide consumption.

[29]For an extensive treatment on empirical evidence on wage rigidity, see Chapter 9.4 of Uribe and Schmitt-Grohé (2017).

[30]I choose the average MPC in that decile.

[31]For instance, 30-30 means that the transfer is paid by the top 30% and paid out to the bottom 30%.

Figure 2.3: **IRF to a transfer shock in the TANK model with flexible wages.**



Note: Here, the transfer shock redistributes 1% of st.st. output from segment 1 to segment 2.

sidered are budget-neutral, I find the "classical" definition of the NPV multiplier to be more informative, but I provide the other measure as well in order to be able to compare my results to the ones obtained by Giambattista and Pennings (2017).

Symmetric redistributions

First, I analyze symmetric redistributions, i.e., redistributions between equally-sized subgroups. Figure 2.4 shows aggregate responses to a redistribution of 1 % steady-state output from the top 50% to the bottom 50%. The redistribution is mildly expansive, with an impact response of output of about 0.14% of its steady-state level (see table 2.2, row "50-50"). Figure 2.5 shows the IRF of aggregate consumption and of segment-specific consumption levels. Note that segments one to five pay the (uniformly distributed) transfer and segments six to ten receive it. As expected from the analysis of the two-segment case, the transfer has a stronger impact on segment-specific consumption, the higher the MPC in that segment.

For a more targeted redistribution from the top 30% to the bottom 30%, the multiplier is about one third larger (see table 2.2, row "30-30"). This could be expected from the considerations of the two-segment case: The two top segments now have to give up relatively more (as they pay in total 1% of steady-state GDP), while the two bottom segments receive more (they get in total 1% of steady-state GDP), compared to the former case. This means that this redistribution will result in a stronger increase of output, as a larger amount is taken from a sub-population with a lower average MPC and transferred to a group with a higher average MPC. Table 2.2 also shows the multipliers of a redistribution from

Table 2.2: **Multipliers of different redistributions.**

Type of redistribution	**Impact**	**Avrg. after x yrs.**			**NPV after x yrs.**			**NPV**
		5	10	20	5	10	20	(Uhlig, 2010)
50-50	0.14	0.10	0.07	0.05	0.57	0.78	0.96	0.11
30-30	0.20	0.14	0.11	0.07	0.81	1.12	1.38	0.16
10-10	0.28	0.19	0.15	0.10	1.13	1.56	1.91	0.22
30-70	0.13	0.09	0.07	0.05	0.53	0.73	0.90	0.11
70-30	0.15	0.11	0.08	0.05	0.63	0.87	1.07	0.12
10-90	0.12	0.09	0.07	0.04	0.50	0.69	0.86	0.10
90-10	0.19	0.13	0.10	0.06	0.76	1.04	1.27	0.15
10-30	0.22	0.15	0.12	0.08	0.89	1.23	1.52	0.18
30-10	0.26	0.18	0.14	0.09	1.05	1.44	1.77	0.21

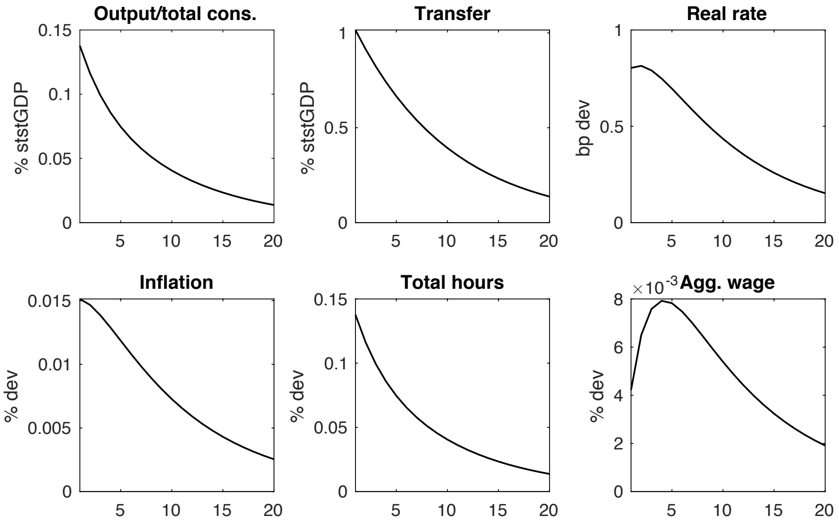
Note: Multipliers are expressed in % of st.st. output.

the top ten percent to the bottom ten percent (row “10-10”). The impact multiplier is twice as large as the one of the 50-50 redistribution. The first three rows of table 2.2 show that for a symmetric redistribution the size of the expansion increases with the degree of targeting of the redistribution: The smaller the top group from which the transfer is taken and the smaller the bottom group which receives the transfer, the larger the impact. This makes intuitive sense, since the same level of redistribution has to be financed by a smaller sub-group, which features relatively less RoT consumers, and is received in turn by a smaller sub-group with relatively more RoT consumers. The redistribution has therefore more favorable implications for aggregate demand.

Asymmetric redistributions

Next asymmetric redistributions are considered, i.e., redistributions from one group to a smaller or larger group. In general, the observation that the multiplier increases in the degree of targeting carries over from the symmetric case. This is due to an increase in aggregate consumption brought about by transferring funds from a subgroup of the population with a lower average MPC to a subgroup with a higher average MPC. However, with asymmetric redistributions, not only the degree of overall targeting, but also the split matters. For example, see rows four and five of table 2.2. Even though the degree of targeting is the same (100% of the population is affected for both splits), the redistribution is more expansive for the “70-30” split than for the “30-70” split. This is because of the non-linear relationship between cash-on-hand and MPC, which resembles a hockey stick, i.e., the negative slope of the MPC as a function of cash-on-hand is steepest for the poorest decile and remains approximately constant for deciles three to ten. That is, when we consider a redistribution that targets a smaller group at the bottom, the aggregate demand effects will be more expansionary, because the average MPC will be higher. The “70-30” is most comparable to the experiment in Giambattista and Pennings (2017),

Figure 2.4: **IRF to a transfer shock in the full model.**



Note: Here, the transfer shock redistributes 1% of st.st. output from the top to the bottom half.

since they consider a redistribution from all Ricardian households that make up 70% in their baseline calibration to all RoT consumers. The NPV multiplier obtained here, computed as in Uhlig (2010), is much smaller than Giambattista and Pennings (2017)'s multiplier of 0.25.

2.3.3 Robustness Analyses

One-off shock

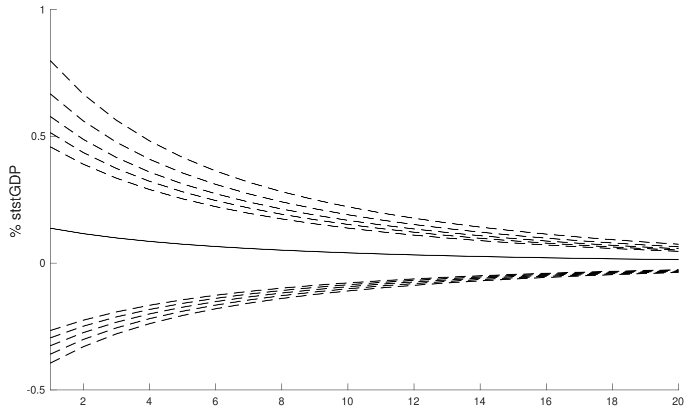
As a first robustness check, I look at a one-off shock that resembles the stimulus payments of the Bush administration in 2001 and 2008, respectively. Table 2.3 depicts the multipliers of the same types of redistributions I have considered before. As the shock is purely transitory, I only depict impact multipliers. As expected, the impact multiplier is slightly larger than in the more persistent case, but the qualitative differences follow closely the ones from before. That is, the impact is larger the more targeted the redistribution, the larger the paying group, and the smaller the receiving group.

Table 2.3: **Impact multipliers of a one-off transfer shock.**

50-50	**30-30**	**10-10**	**30-70**	**70-30**	**10-90**	**90-10**	**10-30**	**30-10**
0.24	0.35	0.49	0.23	0.27	0.22	0.33	0.38	0.46

Note: Multipliers are expressed in % of st.st. output.

Figure 2.5: **Consumption IRF.**



Note: IRF of total consumption (solid line) and segment-specific responses (dashed lines).

An alternative distribution of MPC

In this subsection I consider a different distribution of the MPC, allowing for the presence of wealthy HtM consumers, as suggested by the evidence by Auclert (2019). I implement this by assuming that the two top segments also feature relatively many RoT consumers, however less than the bottom segment. The result is a right-skewed u-shaped distribution. Table 2.3 depicts the impact multipliers for the different redistribution splits I have considered before under the new calibration.

Table 2.4: **Impact multipliers of a one-off transfer shock with wealthy HtM.**

50-50	**30-30**	**10-10**	**30-70**	**70-30**	**10-90**	**90-10**	**10-30**	**30-10**
0.12	0.16	0.20	0.09	0.14	0.04	0.18	0.14	0.22

Note: Multipliers are expressed in % of st.st. output.

As expected, the results differ due to the different direct consumption effects of the redistribution. For instance, the “50-50” redistribution is slightly less expansive than in the case without wealthy HtM consumers (0.12 versus 0.14), as the average MPC in the transfer-paying group is higher than in the benchmark case. The same is true for the “30-30” redistribution (0.16 versus 0.2), and the “10-10” redistribution (0.20 versus 0.28).

Table 2.4 tells us that the biggest “bang for the buck” that we can achieve with top-to-bottom redistributions occurs when redistributing from the top 30% to the bottom 10% with an impact multiplier of 0.22. This could be expected, because the bottom 10% is the subgroup with the highest MPC, while the top 30% is the subgroup with the lowest MPC. However, also a “40-10” and a “50-10” redistribution result in expansions of similar size (not shown in the table), even though the average MPC both in the top 40% and top 50% is higher than that in the top 30%, and hence has smaller direct effects on consumption. However, this is offset by the stronger upward adjustment of labor income.

One obvious candidate to test is a redistribution from the third to the tenth segment, as these are the two segments with the lowest and highest MPC, respectively, and hence trigger the strongest direct effect. This type of redistribution brings about an impact multiplier of 0.24. The second highest expansion can be achieved by a redistribution from segments 2-4 to the bottom segment (with an impact multiplier of 0.23).

Other robustness checks

Table 2.5 shows that all impact multipliers are considerably larger when monetary policy is less responsive ($\phi = 1.1$). When the central bank increases its target rate less aggressively in response to the increase in inflation, which reduces Ricardian consumption by less.[32]

Table 2.5: **Impact multipliers with less responsive monetary policy.**

50-50	**30-30**	**10-10**	**30-70**	**70-30**	**10-90**	**90-10**	**10-30**	**30-10**
0.2	0.28	0.39	0.18	0.22	0.17	0.26	0.31	0.36

Note: Multipliers are expressed in % of st.st. output.

The results are also robust for a finer grid of MPCs. For instance, with $N = 20$ a redistribution from the top 2 segments to the bottom 2 segments (which constitutes a "10-10" redistribution) generates an impact multiplier of 0.28, the same as in the $N = 10$ case.

2.4 Conclusion

In this paper, I analyze the role of MPC heterogeneity for the effectiveness of redistributive policies. A distribution of MPC is introduced by partitioning the household population into segments with different share of credit-constrained consumers, which allows targeting empirical estimates. In the baseline case, I find that redistributing 1% of steady-state output from the top half to the lower half of the MPC distribution increases output on impact by 0.14% of steady-state output. The the size of the multiplier increases considerably with more concentrated redistribution splits. Qualitatively, the expansion is driven by two adjustment mechanisms. First, a Keynesian effect arises as a lump-sum tax is taken from a group of households with a lower MPC and transferred to a group with a higher MPC. Second, employment in the paying group increases by more than employment in the receiving group decreases. The size of the multiplier increases in the degree of targeting of the extremes of the distribution. Finally, for a

[32] Giambattista and Pennings (2017) find the transfer multiplier is much more sensitive to monetary policy than the purchases multiplier, because transfers imply more inflationary pressure.

given degree of targeting, the redistribution is more expansive the larger the paying group relative to the receiving group due to a non-linear relationship between cash on hand and MPC for poor deciles.

Although this ad-hoc approach is subject to the drawback that the distribution of MPC is not endogenously generated, it is a useful approximation to compute quantitatively meaningful redistribution multipliers, and constitutes a way to introduce an empirically plausible MPC distribution in a wide array of DSGE models. It establishes a starting point for future research, which should be aimed at microfounding the MPC distribution that allows us to match its empirical counterpart.

Chapter 3

Household Heterogeneity and the Adjustment to External Shocks

Abstract

This paper analyzes the role of household heterogeneity in the transmission of adverse external shocks. To this end, I build a small-open-economy model where households differ in their holdings of an asset denominated in foreign currency. I find that the shock mainly transmits via adverse effects on households' real income, whereby poor households are hit harder. My results further show that rich households dis-proportionally benefit from the domestic stabilization provided by a devaluation of the nominal exchange rate.

JEL classification: E21, E61, E62.
Keywords: Heterogeneous agents, small open economy, foreign currency debt, exchange rate policy.

3.1 Introduction

Over the last decades many emerging economies have experienced reoccurring external crises, typically characterized by sudden stops of capital inflows and sharp drops in output. During the Great Financial Crisis, also advanced economies faced an abrupt deterioration of external financing conditions. The theoretical literature on the propagation of shocks driving these crises has largely focused on the representative-agent (RA) framework, even though empirical evidence suggests that various characteristics of external crises can have a differential impact across households. (i) Poor households suffer more from a decrease in labor income than rich households (see, e.g., Guvenen et al. (2017) for the US and ECB (2016) for Europe). (ii) Nominal devaluations redistribute from debtor to creditor households when debt is denominated in foreign currency (see, e.g., Drenik et al., 2018). (iii) Poor households spend a relatively larger share of their income on tradable goods (see, e.g., Carroll and Hur, 2020), implying a differentiated impact of terms of trade changes. (iv) The marginal propensity to consume (MPC) depends negatively on wealth (see, e.g., Carroll et al., 2017), which implies that the anti-poor redistributional consequences captured by stylized facts (i)-(iii) have an effect on aggregate dynamics. I develop a heterogeneous-agent (HA) small open economy (SOE) model that captures these key aspects to analyze the transmission of external shocks both at the aggregate level and for households at different income levels.

My model is based on Schmitt-Grohé and Uribe (2016)'s RA-SOE model (SGU (2016) henceforth). There- in, the representative household consumes tradable and non-tradable goods and debt is denominated in foreign currency. Non-tradable goods are produced by perfectly competitive firms whose wage setting is constrained by downward nominal wage rigidity (DNWR). In my HA version of the model, households are subjected to idiosyncratic productivity shocks and face an ad-hoc borrowing constraint (as in Bewley, 1977), which introduces heterogeneity in foreign asset holdings. It allows the model to capture the empirical finding that changes in labor income and in the nominal exchange rate have a differentiated impact on household incomes (capturing stylized facts (i) and (ii), respectively). Moreover, wealth heterogeneity introduces differences in the MPC across households (see stylized fact (iv)). I also present an extension of the model with non-homothetic Stone-Geary preferences for tradable goods, which allows me to capture the empirically observed differences in consumption baskets (see stylized fact (iii)).

I analyze the propagation of two adverse external shocks, a hike in the world interest rate and a negative shock to households' tradable endowment representing a terms-of-trade deterioration.[1] The first set of results concerns the benchmark model with homothetic preferences, so the only aspect in which households differ are their wealth holdings. I show that, as in SGU (2016)'s RA model, a pegged exchange rate, in the presence of DNWR, prevents a real devaluation that would be necessary to stabilize the domestic economy and avert unemployment. In my HA model, however, the adverse shock mainly transmits via negative indirect effects on households' real income, rather than through direct effects on intertemporal substitution.[2] Thereby, real incomes of poor households are affected more adversely

[1]These shocks are considered key drivers of external crises, as, e.g., argued in Uribe and Schmitt-Grohé (2017).

[2]I refer to the direct effect of a shock as the effect on consumption changes induced by the shock only, i.e., holding all other variables constant. The indirect effect is then the effect on consumption that is caused by general-equilibrium adjustments.

under both exchange rate regimes, which is reflected in a steeper fall in final consumption of poor households, used here as a proxy for welfare. High-income consumers benefit dis-proportionally from the stabilization provided by an exchange rate devaluation.

The intuition behind these results is as follows. Wealth heterogeneity matters for both the direct and the indirect effects of the shocks. The direct effect of the interest rate shock on consumption differs across households depending on their asset holdings, since they influence the strength of the income effect and, due to the borrowing constraint, the degree to which intertemporal substitution is at work. A shock in the tradable endowment, on the other hand, affects households differently through the heterogeneous share of the endowment in total income. The direct effects of both shocks lead to a drop in aggregate consumption, which brings about general-equilibrium repercussions that depend on the exchange rate regime in place. Under a flexible exchange rate regime, I show that a depreciation of the nominal exchange rate, aimed at clearing the labor market, has the same effect on households' real income as a decrease in nominal wages. Implied redistributional consequences are anti-poor in the sense that they lower the real income of low-income households the most (compare stylized facts (i) and (ii)). This finding also implies that, as in SGU (2016)'s RA model, a flexible exchange rate policy aimed at clearing the labor market can replicate the outcome with perfectly flexible wages. Under a fixed exchange rate system, households' real income is negatively affected by the decrease in hours worked, which is most detrimental to poor households (compare again stylized fact (i)).

I then present an extension of the model that allows to capture empirical evidence showing that the expenditure share spent on tradable goods is negatively correlated with both income and wealth (compare stylized fact (iii)). To model these non-homothetic preferences, I introduce a Stone-Geary utility function calibrated to match tradable shares both along the income and the wealth distribution. I show that non-homothetic preferences constitute an expenditure-switching friction and imply that poor households need to be incentivized more strongly to switch from tradable to non-tradable consumption. As a consequence for the aggregate adjustment, the real exchange rate devalues more strongly with a flexible exchange rate regime or there is higher unemployment when the exchange rate is pegged. Thereby, under both exchange rate policies, anti-poor redistributional consequences are deepened, compared to the homothetic case. In a last exercise I show that a countercyclical tax can dampen this externality and thus reduce the severeness of the recession.

The paper relates to the literature as follows. My model combines nominal rigidities with idiosyncratic productivity shocks and an ad-hoc borrowing constraint as in Bewley (1977), which is shared by a large literature on aggregate shocks in HA economies.[3] There is a rapidly growing literature on the redistributive consequences of monetary policy shocks and related aggregate effects (see, e.g., Kaplan et al. (2018) or Auclert, 2019).[4] While the shock mainly transmits via the direct impact on intertemporal substitution in the RA New Keynesian (NK) model, the HANK model attributes a greater role to indirect (general-equilibrium) effects that cause redistribution among heterogeneous households, a channel that is also important in my model.[5] Cravino et al. (2020) show that monetary policy shocks can also

[3] Kaplan and Violante (2018) provide an overview of aggregate shocks for which household inequality matters.

[4] Other examples include Farhi and Werning (2019), Luetticke (forthcoming), McKay et al. (2016) and Gornemann et al. (2016).

[5] However, Ottonello and Winberry (forthcoming) show that in models that focus on firm heterogeneity the indirect effect is

redistribute between rich and poor households by having a different impact on the inflation rates they face. This type of redistribution channel is also present in my model when I include non-homothetic preferences. HA models have also been used to study fiscal policy, which can directly influence households' income. Examples include **?**, Oh and Reis (2012), and Kaplan and Violante (2014). Finally, I solve the model using methods developed in Reiter (2009) and Reiter (2010).

While the above mentioned papers are concerned with a closed-economy framework, my research question requires an open-economy setup. A SOE is an obvious choice, since I am interested in the effects of external shocks on the domestic economy, and not in the origins of the shocks. RA models in the SOE literature can be categorized in two branches. One branch features nominal rigidities such as DNWR in SGU (2016) on which my model is based, other examples include Kollmann (2002), Galí and Monacelli (2005), and Galí and Monacelli (2016). A second approach is to abstract from nominal rigidities and instead emphasize the role of financial accelerators, captured by endogenous borrowing constraints (see, e.g. Mendoza, 2010). These papers typically focus on sudden stop dynamics in emerging economies, see also Benigno et al. (2013), Bianchi (2011), Lorenzoni (2008), and Uribe (2006). My paper features elements of both strands in that it combines nominal rigidities with a borrowing constraint, which is however ad-hoc. There are only a few papers analyzing the propagation of external shocks in a HA setup. Iyer (2015) and Cugat (2019) employ two-agent NK models to analyze the differential impact of indirect effects of external shocks on labor income. Their analyses remain, however, silent on heterogeneous revaluation and consumption-basket effects. Mendoza et al. (2007) is an early example of a fully-fledged HA open-economy model used to analyze a permanent shock to the interest rate. Finally, De Ferra et al. (2020)'s analysis of Hungary's sudden stop in the late 2000s is most closely related to my paper: It shares both the HA setup and the focus on differentiated indirect effects. There are two important differences, however. First, they do not analyze the economy's adjustment to typical exogenous drivers of external crises (such as interest-rate and terms-of-trade shocks), but instead consider a series unexpected shocks that "force" the current account to reverse. Second, they focus on a comparison between a nominal devaluation and an adjustment via wages and prices in a model where households have access to a second asset denominated in domestic currency. They show that in this case the equivalence result no longer holds.

My paper also adds to the relatively new literature on the consequences of heterogeneous consumption baskets for short-term dynamics.[6] Cravino et al. (2020) use their estimates of the income-dependency of consumption baskets to calibrate this heterogeneity in an NK model, in which it is introduced in an ad-hoc manner. In a similar ad-hoc setup, Clayton et al. (2018) analyze the implications of consumption basket heterogeneity for the propagation of a monetary policy shock. My paper differs from Cravino et al. (2020) and Clayton et al. (2018) in that non-homothetic relative demand arises endogenously from Stone-Geary preferences. The resulting negative correlation between inflation rates and the MPC is crucial to capture consumption effects, which is essential for my research question.

much less important than in heterogeneous-household models.

[6]Traditionally, in the macroeconomic literature, non-homothetic preferences have only been employed to investigate changing sectoral compositions, as countries grow richer. For some classic references on structural change, see, e.g., Matsuyama (1992), Laitner (2000), Kongsamut et al. (2001), or, more recently, Boppart (2014).

The remainder of the paper is organized as follows. Section 3.2 lays out the HA-SOE model. In section 3.3, I analyze the transmission mechanism under a flexible and a fixed exchange rate regime. Section 3.4 considers the role of non-homothetic preferences. Section 3.5 concludes.

3.2 A Small Open Economy with Heterogeneous Households

The model is a heterogeneous-household version of SGU (2016)'s SOE model. Households self-insure against idiosyncratic shocks by acquiring internationally traded assets on which they earn an interest at a rate that is exogenous to the SOE. Assets are denominated in foreign currency and borrowing is subject to an ad-hoc constraint. There is an endowment of tradable goods, while non-tradable goods are produced domestically.

3.2.1 Households

There is a unit mass of infinitely-lived ex ante homogeneous households. Households maximize expected life-time utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t).$$

Parameter $0 < \beta < 1$ is the subjective discount factor and expectations are conditional on the information available in period 0, which includes beginning-of-period wealth a_0 and the realization of an idiosyncratic income shock s in period 0. Consumption c_t is a composite of tradable goods c_t^T and non-tradable goods c_t^N

$$c_t = A(c_t^T, c_t^N), \tag{3.1}$$

where $A(...)$ is an aggregator that is an increasing, concave, and linearly homogeneous function.

Households inelastically supply $\bar{h}$ units of labor. Hence, for hours worked it holds that $h_t \leq \bar{h}$. Labor productivity is subjected to idiosyncratic shocks s_t, which follow a first-order Markov chain with N_s possible realizations. The associated conditional transition probability is given by

$$\pi\left(s' \mid s\right) = Prob\left\{ s_{t+1} = s' \mid s_t = s\right\}. \tag{3.2}$$

A unit of effective labor $s_t h_t$ earns wage W_t, taken as given by households. Households have access to an internationally traded, state non-contingent bond a_t, which is denominated in foreign currency. The period budget constraint is given by

$$P_t^T c_t^T + P_t^N c_t^N + \mathcal{E}_t a_{t+1} = P_t^T y_t^T + \mathcal{E}_t a_t (1+r) + s_t W_t h_t + \phi_t, \tag{3.3}$$

where P_t^T and P_t^N denote the price of tradables and non-tradables, respectively, both taken as given by households. The nominal exchange rate $\mathcal{E}_t$ is defined as the domestic-currency price of one unit of foreign currency, r_t denotes the exogenous real world interest rate, and y_t^T is the household's endowment of the tradable good. Households own the firm that produces the non-tradable good and profits

ϕ_t are distributed uniformly across the population. It is assumed that each household has the same endowment of tradable goods for which the Law of One Price holds

$$P_t^T = \mathcal{E}_t P_t^{T*}, \tag{3.4}$$

where P_t^{T*} denotes the price of the tradable good in foreign currency. Normalizing $P_t^{T*} = 1$ implies that the price of tradables is equal to the nominal exchange rate, $P_t^T = \mathcal{E}_t$, so one can rewrite the budget constraint as

$$c_t^T + p_t c_t^N + a_{t+1} = y_t^T + a_t(1 + r_t) + s_t w_t h_t + \frac{\phi_t}{\mathcal{E}_t}, \tag{3.5}$$

where $p \equiv \frac{P^N}{P^T}$ is defined as relative price of non-tradables and $w \equiv \frac{W}{\mathcal{E}}$ is the real wage. Debt is subject to an ad-hoc borrowing constraint

$$a_{t+1} \geq \underline{a}. \tag{3.6}$$

Associated optimality conditions of the household problem are

$$p_t = \frac{A_2(c_t^T, c_t^N)}{A_1(c_t^T, c_t^N)}, \tag{3.7}$$

$$\lambda_t = U'(c_t) A_1(c_t^T, c_t^N), \tag{3.8}$$

$$\lambda_t = \beta \left(1 + r_t\right) \lambda_{t+1} + \mu_t, \tag{3.9}$$

$$\mu_t \left(a_{t+1} - \underline{a}\right) \geq 0, \tag{3.10}$$

where λ_t is the Lagrange multiplier associated with the budget constraint and μ_t the one for the borrowing constraint.[7] Equation (3.7) describes the relative demand for non-tradable goods for a given level of tradable goods, depicted in figure 3.1. Here, tradables act as a demand shifter, i.e., given an increase in the demand for tradables, relative demand for non-tradables increases, ceteris paribus.

3.2.2 Firms

The supply side is highly stylized, as the focus of this paper is demand-side heterogeneity. Perfectly competitive firms produce non-traded output and use aggregate labor L_t as the only input for the production of the non-tradable good. The profit maximisation problem of a firm is given by

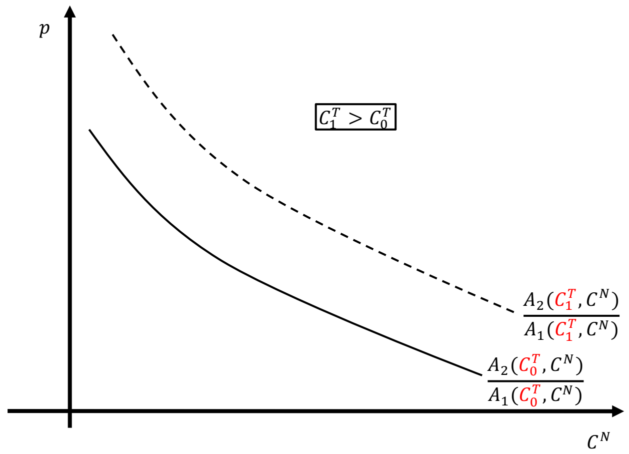
$$\max_{L_t} \Phi_t = P_t^N Y_t^N - W_t L_t, \tag{3.11}$$

where the production of non-tradable output follows a concave production function $Y_t^N = F(L_t)$. The associated optimality condition is

$$P_t^N F'(L_t) = W_t \Leftrightarrow p_t F'(L_t) = \frac{W_t}{\mathcal{E}_t}. \tag{3.12}$$

[7] A_1 (A_2) denotes the partial derivative w.r.t. the first (second) argument.

Figure 3.1: **Demand for non-tradable goods.**



Note: The presentation follows SGU (2016). The representation in terms of aggregate variables C^T and C^N is admissible because all households have the same homothetic preferences, so equation (3.7) holds also w.r.t. to aggregate quantities.

Using the production function, we can derive

$$p_t = \frac{W_t/\mathcal{E}_t}{F'(F^{-1}(Y_t^N))}, \tag{3.13}$$

which describes the supply schedule of non-tradable goods for a given wage and a given nominal exchange rate. Figure 3.2 shows how nominal wage and nominal exchange rate shift supply.

3.2.3 Shocks

The model features interest-rate and terms-of-trade shocks, both of which are considered key drivers of business cycles in emerging economies. The exogenous process for the world interest rate follows

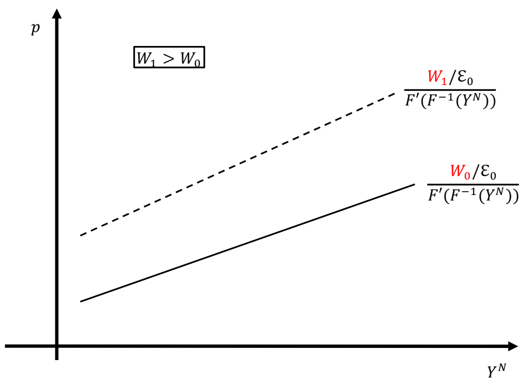
$$\ln\left(\frac{1+r_t}{1+r}\right) = \rho_r \ln\left(\frac{1+r_{t-1}}{1+r}\right) + \epsilon_t^r, \tag{3.14}$$

where the i.i.d. zero-mean shock ϵ_t^r (with standard deviation σ^r) causes the interest rate to fluctuate around its steady-state value r with a persistence governed by $0 < \rho_r < 1$. SGU (2016) point out that shocks to the tradables endowment y^T can be interpreted as terms-of-trade shocks, as they affect households' purchasing power in the same way as a shock to the relative price of domestic and foreign tradables. They evolve according to

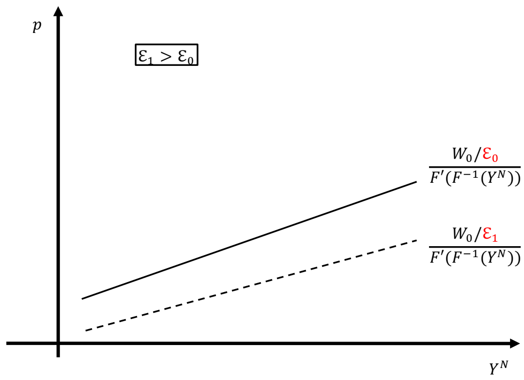
$$\ln\left(\frac{y_t^T}{y^T}\right) = \rho_{y^T} \ln\left(\frac{y_{t-1}^T}{y^T}\right) + \epsilon_t^{y^T}, \tag{3.15}$$

Figure 3.2: **Supply of non-tradable goods.**

(a) A increase in the wage.



(b) An increase in the nominal exchange rate.



Note: The presentation follows SGU (2016).

where y^T is the steady-state value of tradable endowment, $\epsilon_t^{y^T}$ is an i.i.d. shock with zero mean and standard deviation σ^{y^T}. Parameter $0 < \rho_{y^T} < 1$ governs the persistence of the endowment's deviations from its stead-state value.

3.2.4 Downward Nominal Wage Rigidity

A collapse in the demand for non-tradable goods, following an adverse external shock, brings about a decrease in labor demand. In the model as presented so far, the labor market always clears by downward adjustment of the nominal wage. However, in reality, nominal wage decreases are very rare due to political economy constraints.[8] To capture this key rigidity, perfect DNWR is imposed by[9]

$$W_t \geq W_{t-1}. \tag{3.16}$$

Ruling out wage reductions leaves exchange rate devaluations as the only remaining adjustment margin to restore labor demand in the face of an adverse shock. Thus, when the nominal exchange rate is fixed because a country pegs its currency or shares it with trading partners, adverse shocks can push labor demand L_t below aggregate labor supply $\bar{L}$ and thereby cause involuntary unemployment. Formally this is the case when

$$L_t \leq \bar{L}, \tag{3.17}$$

with

$$\bar{L} = \bar{s}\bar{h}, \tag{3.18}$$

where $\bar{s}$ is average labor productivity.[10] Accordingly, the complementary slackness condition is

$$(\bar{L} - L_t)(W_t - W_{t-1}) = 0, \tag{3.19}$$

[8] See SGU (2016) for an extensive summary of empirical evidence on DNWR.
[9] SGU (2016) consider non-perfect downward wage rigidity with $W_t \geq \kappa W_{t-1}$, where $\kappa = 0.99$.
[10] Average labor productivity is constant over time due to the law of large numbers.

i.e., when equation (3.16) is binding, labor will be demand-determined, otherwise it is supply-determined. In the following I assume that, when involuntary unemployment arises, individual households work less hours then their inelastic labor supply. That is, each household experiences some involuntarily unemployed hours $h_t < \bar{h}$, where $L_t = \sum_a \sum_s s\ h_t\ \psi_t(s,a)$ and $\psi_t(s,a)$ is the distribution of households over individual states (s,a).

3.2.5 Exchange Rate Policy

Two different exchange rate regimes are considered. The first regime with a fixed exchange rate ($\mathcal{E}_t = \bar{\mathcal{E}}\ \forall t$) represents countries with a currency peg (adopted in many emerging economies) or those belonging to a currency union within which most trade occurs. Under the second regime, the central bank can freely adjust $\mathcal{E}_t$ and is assumed to follow two different rules depending on whether or not DNWR is binding. When DNWR is not binding and wage adjustment can ensure that the labor market is cleared, $\mathcal{E}_t$ is left constant. When DNWR is binding, the central bank maintains labor market clearing by adjusting the nominal exchange rate. Formally, it sets

$$\mathcal{E}_t^o = \frac{\bar{W}}{\frac{A_2(C_t^T, F(\bar{L}))}{A_1(C_t^T, F(\bar{L}))} F'(\bar{L})}. \tag{3.20}$$

if $W_t = W_{t-1} = \bar{W}$. Exchange rate policy $\mathcal{E}_t^o$ induces the labor-market-clearing equilibrium by equating demand for non-tradables (defined in equation (3.7)) with its supply (given in equation (3.13)). SGU (2016) show that this policy can replicate the Pareto-optimal allocation of the frictionless economy in their RA model.[11] Intuitively, the allocation where households are able to work all of of their inelastically supplied hours also maximizes their final consumption.[12] This allocation is likely to be Pareto-optimal in a HA setup as well, but a formal proof is beyond the scope of this paper.

3.2.6 Equilibrium

The economy consists of four markets: the asset market, the labor market, the markets for tradable and non-tradable goods. Regarding the asset market, the model resembles a Huggett (1993) economy in that there is no domestic asset supply as production uses labor as the only input. Since the world interest rate is in general not equal to the closed-economy equilibrium interest rate r^*, households' individual asset positions do not balance out and the economy's aggregate net foreign asset position (NFA) differs from zero. The labor market does not clear if the wage rigidity condition is binding and the exchange rate is fixed. The market for tradable good always clears by assumption, since the trade balance adjusts in response to changes in the net foreign asset position (see appendix A.2.2). Finally, the relative price for non-tradables adjusts to equate demand and supply of non-tradables. Formally, an equilibrium is then defined as follows.

[11] In the RA model, households do not face idiosyncratic shocks and are only constrained by the natural borrowing limit. The Pareto-optimal allocation results from the choice of a benevolent local social planner who is not constrained by nominal rigidity, but takes as given the international market structure and domestic production possibilities.

[12] I refer the reader to SGU (2016) for the formal derivation of the planner's problem.

Definition 1: *Given stochastic processes* $\{r_t\}$ *and* $\{y_t^T\}$*, borrowing constraint* $\underline{a}$*, and an initial joint distribution over individual states* $\psi_0(s,a)$*, the equilibrium of this economy is a sequence of prices* $\{p_t^N, w_t\}$*, value functions* $\{V_t(s,a)\}$*, individual decision rules* $\{c_t^T(a,s), c_t^N(a,s), a_{t+1}(a,s)\}$*, and aggregate labor* $\{L_t\}$ *such that:*

- $c_t^T(a,s)$, $c_t^N(a,s)$, *and* $a_{t+1}(a,s)$ *solve the household problem.*
- *The sequence of distributions* $\{\psi_t(s,a)\}$ *is consistent with the initial distribution, individual policy functions, and the idiosyncratic shocks.*
- *The Law of One Price holds.*
- *Firms maximize profits, so the wage is set according to (3.12).*
- *Labor supply is consistent with equations (3.16), (3.17), (3.18), and (3.19).*
- *The market for non-tradable goods clears:* $\sum_a \sum_s c_t^N(a,s)\ \psi_t(s,a) = F(L_t)$.

Using these equilibrium conditions, we can solve for several other variables of interest following Uribe and Schmitt-Grohé (2017).[13] The real exchange rate can be derived as

$$RER_t = A_1(G^{-1}(p_t), 1), \tag{3.21}$$

where A_1 denotes the partial derivative w.r.t. the first argument, and function G is defined as $p_t = \frac{A_2(C_t^T, C_t^N)}{A_1(C_t^T, C_t^N)} \equiv G(\frac{C_t^T}{C_t^N})$ with $C_t^T \equiv \sum_a \sum_s c_t^T(a,s)\ \psi_t(s,a)$ and $C_t^N \equiv \sum_a \sum_s c_t^N(a,s)\ \psi_t(s,a)$ defined as aggregate demand for tradables and non-tradables, respectively. The net foreign asset position, expressed in terms of tradable goods, is equivalent to net aggregate asset demand

$$NFA_{t+1} = \sum_a \sum_s a_{t+1}(a,s)\ \psi_t(s,a) \tag{3.22}$$

and the current account in terms of tradables can be derived as

$$CA_t = TB_t - r_t NFA_t, \tag{3.23}$$

where TB_t is the trade balance expressed in tradable goods.

3.2.7 Calibration and Functional Forms

The model is calibrated to the Argentinean economy at a quarterly frequency. My calibration strategy is to set certain parameters according to empirical evidence provided in SGU (2016) (see references therein for more details) and to choose the remaining ones to match long-term averages.

Production in the non-tradable sector follows $F(L) = L^\alpha$, where the labor share α is set to 0.75 following SGU (2016). The individual inelastic labor supply is normalized to $\bar{h} = 1$. The utility function is of standard CRRA form, $U(c) = \frac{c^{1-\sigma}}{1-\sigma}$, with σ being the coefficient of relative risk aversion. Consumption c is aggregated via a CES technology, $A(c^T, c^N) = \left[\gamma \left(c^T\right)^{1-\frac{1}{\varepsilon}} + (1-\gamma)\left(c^N\right)^{1-\frac{1}{\varepsilon}}\right]^{\frac{1}{1-\frac{1}{\varepsilon}}}$.

[13] For the derivations, see appendix A.2.2.

Table 3.1: **Calibration of model parameters.**

Parameter		**Value**	**Source/Target**
Relative risk aversion	σ	2	
Elasticity of substitution	ε	0.5	
St.st. quarterly interest rate	r	0.03	Schmitt-Grohé
Labor share in non-traded sector	α	0.75	and Uribe (2016)
Labor endowment	$\bar{h}$	1	
Autocorr. interest rate shock	ρ_r	0.86	
Autocorr. terms-of-trade shock	ρ_{y^T}	0.41	Uribe and Schmitt-Grohé (2017)
Autocorr. idios. shock	ρ_s	0.9	Storesletten et al. (2004)
Std. dev. idios. shock	σ_u	0.2	
CES weighting factor	γ	0.26	$C^N/Y^N = 1$
St.st. endowment of tradables	y^T	0.55	$Y^T/Y = 0.32$
Discount factor	β	0.95	Debt-to-GDP ratio = 0.23
Borrowing constraint	$\underline{a}$	-3.5	Wealth Gini of 0.79

Relative risk aversion σ is set to 2 and the elasticity of substitution between tradable and non-tradable consumption ε to 0.5, which are both standard values and in line with existing empirical evidence (see again SGU, 2016). The autocorrelation coefficient of the interest rate shock is set to 0.86 as in SGU (2016). The shock to tradable endowment follows an AR(1) process with an autocorrelation coefficient of 0.41 (see Uribe and Schmitt-Grohé, 2017).

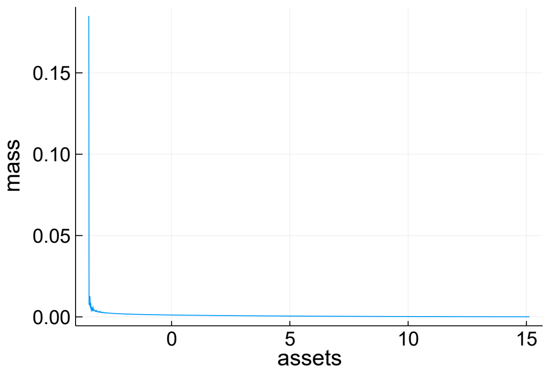
The idiosyncratic labor productivity follows an AR(1) process which is discretized as a Markov chain with 10 states using the Tauchen method. It is calibrated to have an autocorrelation of 0.9 and a standard deviation 0.2, both of which are standard values in the literature on the estimation of idiosyncratic productivity shocks, see, e.g., Storesletten et al. (2004). The CES weighting factor γ is set to ensure that aggregate non-tradable consumption is equal to aggregate non-tradable output in steady state.

The rest of the parameters are calibrated to replicate long-term averages of the Argentinean economy in the steady state of the model. Endowment of tradables is 0.55 to target the ratio $\frac{Y^T}{Y} = 0.23$, a target taken from Bianchi (2011). The discount factor β is set to 0.955 to get an external-debt-to-GDP ratio of 0.23 as indicated in SGU (2016). Finally, the borrowing constraint is set to -3.5 to reproduce a wealth Gini of 0.79, which is an estimate from Credit Suisse (2018) for Argentina in 2018.

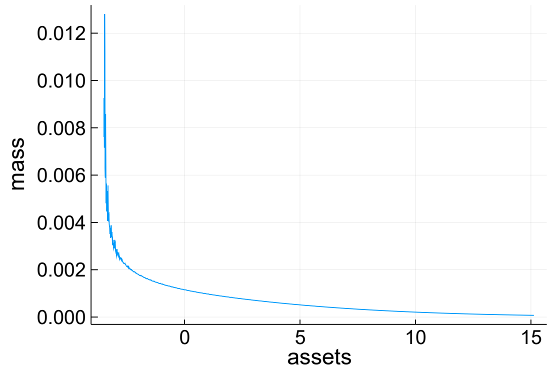
The upper two panels of figure 3.3 depict the resulting steady-state distribution of net foreign assets $\psi(s, a)$. The left panel depicts the distribution including the borrowing constraint, while the right panel excludes it for better readability. Inequality in net foreign asset holdings is relatively high, in accordance with the high Gini coefficient of 0.79. The share of borrowing constrained consumers is, at around 18%, at the lower end of estimates and implies a conservative calibration of MPC heterogeneity, which

Figure 3.3: **Steady-State NFA distribution.**

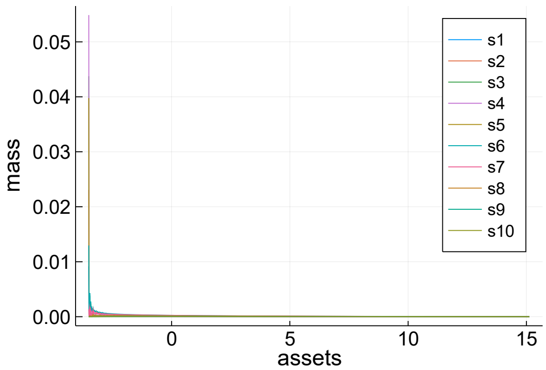
(a) Cumulative distribution.



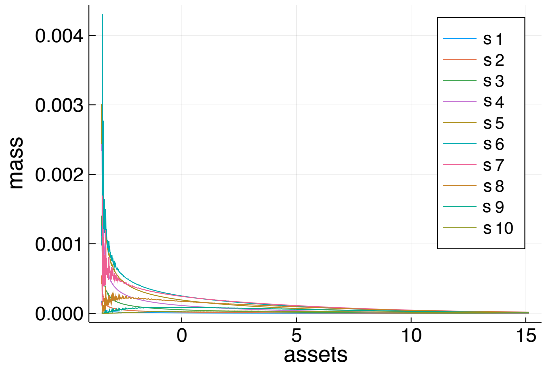
(b) Cumulative distribution
(excluding borrowing constraint).



(c) Distributions for each income realization.



(d) Distributions for each income realization
(excluding borrowing constraint).



Notes: The upper row panels depict the total distribution of net foreign assets in steady state, $\psi(s, a)$. The left panel shows the distribution with the x-axis including the borrowing constraint, while the right panel excludes the borrowing constraint. The two lower panels show the net foreign asset distribution conditional on a specific productivity shock realization, $\psi(s = s_j, a),\ j = 1, ..., 10$, where s_1 is the lowest realization and s_{10} is the lowest.

is a crucial determinant of shock transmission.[14] The two lower panels (the left panel including the borrowing constraint and the right panel excluding it) show the asset distribution conditional on a specific productivity shock realization, where s_{10} is the highest realization and s_1 the lowest. There is a high correlation between asset holdings and shock realizations, as shock persistence makes it likely to have a series of similar realizations that accumulates to high or low asset holdings. As a consequence, low-income households are more likely to be borrowing-constrained than high-income households and anti-poor redistributional consequences have a negative impact on aggregate demand.

3.2.8 Solution

The model is solved by a first-order perturbation in aggregate shocks around a stationary steady state that is fully non-linear in idiosyncratic shocks (as developed in Reiter, 2009), in combination with almost-exact state aggregation (see Reiter, 2010). I solve the household problem on a discrete grid. The associated value function has 300 grid points in the asset dimension and ten grid points in the productivity dimension. When approximating the asset distribution, I use 1001 grid points in the asset dimension and ten grid points in the productivity dimension, which results in an aggregate state space of 10010 variables. The loss-less state reduction shrinks the state space to 345 variables.

3.3 The Transmission Mechanism

In this section I employ the benchmark model with wealth inequality and homothetic preferences to analyze the transmission of shocks to the world interest rate and to the tradable goods endowment. To this end, I first provide intuition on the role of wealth inequality in the shock transmission. The second subsection reports numerical results comparing a flexible and a fixed exchange rate regime.

3.3.1 The Role of Wealth Inequality

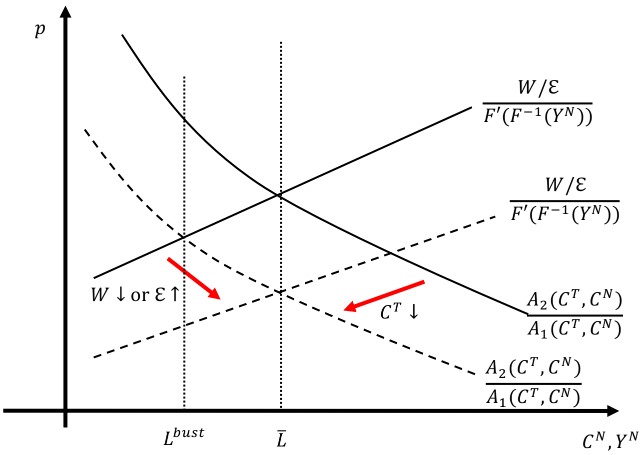
This subsection provides intuition on how the shock transmission is affected by inequality in net foreign asset holdings. The first part focuses on the heterogeneous impact of the shocks' direct effects. I then analyze how indirect effects differ across the two considered exchange rate regimes.

In order to understand the role of wealth inequality in the shock transmission, it is instructive to briefly recap the transmission mechanism in an RA model.[15] To this end, figure 3.4 provides a graphical analysis of the equilibrium adjustments in the market for non-tradables. The direct effects of both adverse external shocks lead to a decline in tradable consumption and shift the demand schedule to the left. For the hike in the interest rate, this follows from intertemporal substitution, and for the shock to tradable endowment it follows from a reduction in household income. To maintain the initial level of non-tradable output and thereby labor market clearing under fixed labor supply, the relative price of non-tradables p has to fall so that expenditure switching offsets the reduction in domestic consumption

[14] See Mankiw (2000) who estimates rule-of-thumb consumers to make up half of the population.

[15] For more details, see SGU (2016).

Figure 3.4: **Adjustment to an adverse external shock.**



Note: The presentation follows SGU (2016).

demand. This can happen either via a reduction in the nominal wage or via a devaluation of the nominal exchange rate, both resulting in a lower real wage w_t and thus higher labor demand and production. Involuntary unemployment – a situation when the labor market does not clear as labor demand falls short of supply $\bar{L}$ – arises if DNWR makes a wage reduction infeasible and an exchange rate peg at the same time rules out a nominal devaluation. A constant real wage then prohibits the equilibrium price reduction that would otherwise restore labor demand from L^{bust} to $\bar{L}$.

In the following, I provide intuition on the role of wealth heterogeneity in the shock transmission. To this end, it is instructive to decompose the total effect of the shock into its direct effect and implied indirect (general-equilibrium) effects and to consider the role of wealth heterogeneity for each of these effects separately. Aggregate tradable consumption C^T is a function of shocks r_t and y_t^T, relative prices w_t and p_t, and exchange rate policy $\mathcal{E}_t$, $C_t^T = C_t^T(\{r_t, y_t^T, w_t, L_t, p_t, \mathcal{E}_t\}_{t\geq 0})$. We can decompose the total effect of an interest rate shock in $t = 0$ by taking the total differential as follows

$$dC_0^T = \sum_{t=0}^{\infty} \frac{\partial C_0^T}{\partial r_t} dr_t + \sum_{t=0}^{\infty} \left(\frac{\partial C_0^T}{\partial w_t} dw_t + \frac{\partial C_0^T}{\partial L_t} dL_t + \frac{\partial C_0^T}{\partial p_t} dp_t + \frac{\partial C_0^T}{\partial \mathcal{E}_t} d\mathcal{E}_t \right) \tag{3.24}$$

where dr_t represents the direct effect of the sock and dw_t, dL_t, dp_t and $d\mathcal{E}_t$ depict implied general-equilibrium adjustments. The change in C_0^T to a shock in the endowment of tradables can be analogously represented by

$$dC_0^T = \sum_{t=0}^{\infty} \frac{\partial C_0^T}{\partial y_t^T} dy_t^T + \sum_{t=0}^{\infty} \left(\frac{\partial C_0^T}{\partial w_t} dw_t + \frac{\partial C_0^T}{\partial L_t} dL_t + \frac{\partial C_0^T}{\partial p_t} dp_t + \frac{\partial C_0^T}{\partial \mathcal{E}_t} d\mathcal{E}_t \right). \tag{3.25}$$

First consider the implications of wealth inequality for the direct effects of an interest rate change on C^T, captured by the first term in equation (3.24). There are three aspects of heterogeneity that cause the direct effect to generally differ from its counterpart in a RA model. First, intertemporal substitution does not apply for borrowing-constrained households and only to a limited extent for those close to the borrowing constraint. Second, the wealth effect implied by the interest-rate shock are heterogeneous across households: Creditor households gain from a higher interest rate, while debtor households face higher debt. With regard to the adjustment of aggregate consumption, this heterogeneity interacts with differences in the MPC, which is the third dimension of relevant heterogeneity. The direct effect of a tradable endowment shock (the first term in (3.25)) differs across households through heterogeneity in the MPC. For borrowing-constrained households a decline in current income translates one-to-one into a reduction in consumption, while non-constrained households smooth out their adjustment.

The indirect effects of the two shocks (the second term in (3.24) and (3.25)) depend on the exchange rate regime in place. I first establish that a flexible exchange rate policy aimed at clearing the labor market can replicate the frictionless adjustment in the face of an adverse shock. This generalizes the result of SGU (2016) who derive it in a RA model. For convenience, I restate the household's budget constraint in real terms

$$c_t^T + p_t c_t^N + a_{t+1} = y_t^T + a_t(1+r_t) + s_t w_t h_t + \frac{\phi_t}{\mathcal{E}_t}. \tag{3.26}$$

Consider a situation in which the real wage, $w_t \equiv \frac{W_t}{\mathcal{E}_t}$, has to decline in response to an adverse shock to maintain a labor market equilibrium without involuntary unemployment. I ask whether the decline in W_t that occurs absent nominal rigidity can be substituted – in the sense of bringing about the same real allocation – by an appropriate exchange rate devaluation when an adjustment of W_t is ruled out. Equivalence between the two alternative shock adjustments (with either W_t or $\mathcal{E}_t$ adjusting) is established by showing that they entail the same changes in individual incomes, and thereby the same change, if any, in the wealth distribution. Starting with real labor income, the change occurring for both adjustments has to be identical as the labor market clears at $\bar{L}$ in both cases. Since both adjustments entail an equal change in real labor income, they must also entail an equal change in real profits. This is because output is linked to the fixed labor supply and therefore also the same in both adjustments.[16] Thus, a shock's impact on individual incomes, and thereby on the real allocation, is independent of whether the adjustment operates via W_t (when wages are flexible) or $\mathcal{E}_t$ (when the central bank substitutes for wage flexibility). This result relies on the assumption that there is just one asset that is denominated in foreign currency.

In the following I compare the outcome when the nominal exchange rates adjusts to replicate the frictionless shock adjustments with the case of a fixed exchange rate. Ex ante, it is not clear that an exchange rate regime that avoids involuntary unemployment (when there is an adverse shock and wages are rigid) improves households' real income, relative to the outcome under a constant exchange rate. When the exchange rate is adjusted to clear the labor market, households experience a decrease in real labor income (due to a decrease in the real wage w_t), while real profit income $\frac{\phi_t}{\mathcal{E}_t}$ stays constant (since

[16] This is apparent in the definition of real profit income $\frac{\phi_t}{\mathcal{E}_t} = p_t y_t^N - \frac{W_t}{\mathcal{E}_t}\bar{h}$. Output y_t^N is the same in both adjustment scenarios because labor supply is the same. Given that the demand schedule shifts in the same way in both scenarios, it follows that p_t has to be identical too. With $p_t y_t^N$ being the same in both adjustments, a given change in the real wage (which is identical by definition of the exchange rate regime) translates into an identical change in real profits.

the decrease in the relative price of non-tradable goods p_t exactly offsets the decreased real labor costs). With a fixed exchange rate, real labor income decreases due to an increase in unemployed hours while profit income declines as non-tradable production is curtailed. In principle, it is possible that the income loss due to the real wage decline necessary to avoid involuntary unemployment outweigh the loss due to involuntary unemployment itself. Household heterogeneity complicates this question, as, e.g., profit and labor income are relatively more important for low-income households who are relatively more adversely affected. In addition, revaluation effects implied by a nominal devaluation hurt debtor households and benefit creditor households, which can further reduce aggregate demand via the heterogeneity in the MPC.

3.3.2 Numerical Results

In this subsection, I numerically compare the adjustment to both adverse shocks under DNWR, once in a scenario with a fixed exchange rate and once when the exchange rate is flexible and adjusts to avoid unemployment.

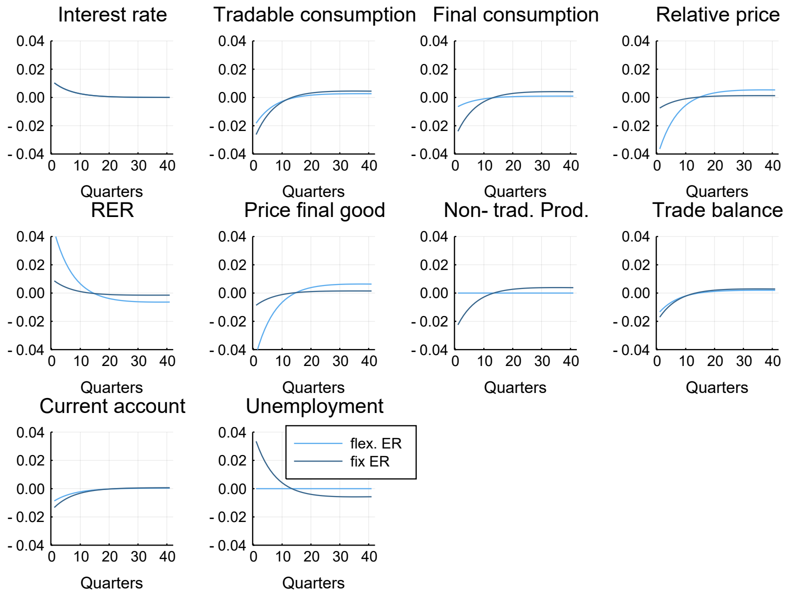
The light blue lines in figure 3.5 show the economy's adjustment to an one-percentage-point increase in the interest rate when the nominal exchange rate is flexible. In reaction to the unexpected hike in the interest rate, households cut back consumption of both types of goods. To stabilize production of the non-tradable good and thereby prevent unemployment, the nominal exchange rate is devalued, bringing about a decrease in real wages and in the relative price of the non-tradable goods or, equivalently, a real devaluation. As outlined in the previous subsection, the exchange rate devaluation redistributes from poor high-MPC households to their rich counterparts, leading to a further drop in aggregate demand and thereby strengthening the need to further devalue. The shock causes the trade balance and the current account to deteriorate, which is only partially offset by the nominal exchange rate adjustment. After approximately five years, the economy stabilizes at a new steady state, which is characterized by a permanently lower real exchange rate and a slightly higher trade balance. As pointed out by SGU (2016), the reason for the permanent effects of the temporary shock is that labor costs have to be permanently lowered to support full employment in the presence of DNWR.

The dark blue lines in figure 3.5 depict the dynamics for an exchange rate peg. The drop in demand for non-tradable goods then causes considerable unemployment and a decline in the production of non-tradable goods. As production falls by less than demand for non-tradable goods, the relative price of non-tradables drops, mirrored by a devaluation of the real exchange rate, which is, however, much smaller than in the case of a flexible nominal exchange rate. The trade balance and the current account deteriorate by more than under the flexible exchange rate case, indicating that also with HA, the combination of DNWR and a peg amplifies an external shock.

In order to judge whether households are better off in a flexible or a fixed exchange rate regime, it is instructive to compare the corresponding responses of final consumption, which is taken here to serve as a proxy for welfare.[17] As stated in SGU (2016), in the RA version of the model adjustments in tradable consumption are the same for a fixed and a flexible exchange rate regime, since households are

[17] An economically meaningful welfare analysis is not possible due to linearity in aggregate shocks of the model solution.

Figure 3.5: **Adjustments to an interest rate shock.**

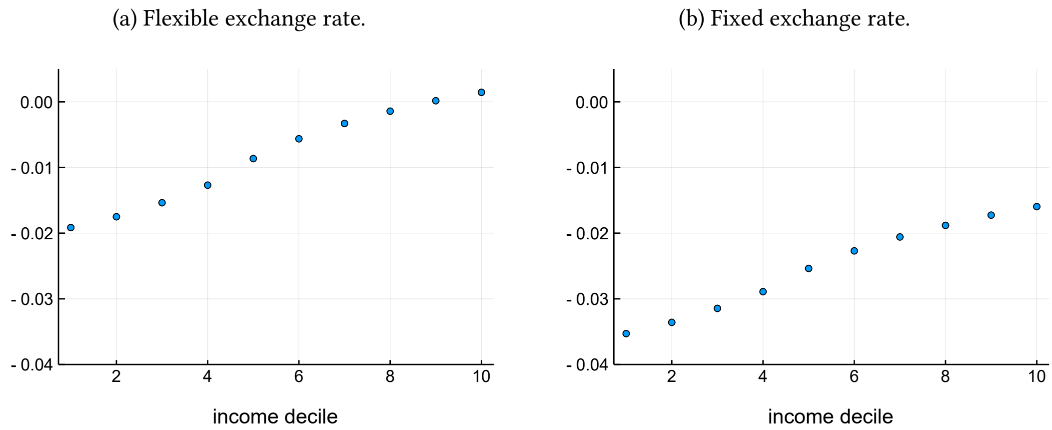


Note: Impulse responses are given in percentage deviations from steady state, except for the interest rate, unemployment, the trade balance and the current account, which are given in absolute deviations.

homogeneous and preferences are additively separable. As non-tradable output is equal to non-tradable consumption, which decreases under the fixed exchange rate policy and stays constant when the exchange rate is flexible, the final consumption of the representative household is significantly lower with a fixed exchange rate. This result is reinforced by household heterogeneity. Here, the dependency of a shock's redistributive implications on the exchange rate regime fosters the decline in tradable consumption under a fixed exchange rate (as will be shown in the next paragraph) and thereby the reduction in final consumption. This indicates that also with HA, the welfare costs associated with unemployment are higher than those of a lower real wage.

Impact responses of final consumption at the income-decile level reveal the redistributive consequences of the two exchange rate regimes and are shown in figure 3.6. Under both regimes, households in the lowest income decile are hit hardest, reflected by the sharpest drop in final consumption, while consumption declines the least for the highest income realization. The redistributive consequences of the shock are thus anti-poor under both regimes, but there are important differences. When the exchange rate is flexible, the positive income effect dominates for households with high asset holdings, so the top income decile even increases consumption. With a fixed exchange rate policy, on the other hand, adverse indirect effects dominate at all income levels. Comparison between the two panels also shows that there is a slightly greater dispersion of consumption responses when the exchange rate is flexible. This means that households in the top deciles benefit dis-proportionally from the stabilization

Figure 3.6: **Impact responses of consumption along the income distribution.**



Note: Impact responses are given in percentage deviations from steady state.

of the domestic economy.

Figure 3.7 shows the economy's adjustment to a ten-percentage-point decrease in tradable endowment. Comparison with figure 3.5 shows that the shock has much more persistent effects than the interest rate, but that qualitatively the results are the same. Hence, in the following analyses, I only report results for the interest rate shock.

3.4 Non-Homothetic Preferences

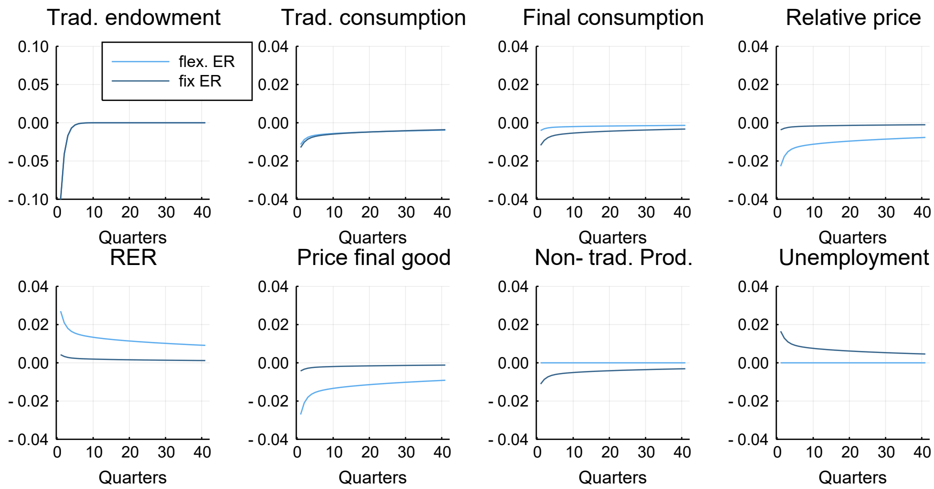
In this section, I expand the model by demand non-homotheticities in the relative demand for tradable goods, which, in interaction with wealth heterogeneity, introduces heterogeneity in consumption baskets. The first subsection reviews empirical evidence serving as a target to calibrate a Stone Geary utility function used to generate non-homotheticity. In the second subsection, I analyze how these non-homothetic preferences impact the economy's adjustment. The last subsection studies how a countercyclical tax can act as an automatic stabilizer.

3.4.1 Model with Non-Homothetic Preferences

This subsection summarizes the empirical evidence on the negative correlation between the share of income spent on tradable goods and household income and wealth.

The seminal work of Engel (1857) establishes that the composition of the consumption basket is very different for poor and rich consumers. In particular, he shows that the share of income spent on food decreases as income rises. Houthakker (1957) is able to confirm this pattern for a broader category of tradable goods and for many countries. While this early evidence focuses on changing consumption behavior as countries grow richer, more recent evidence finds the same correlation for a cross-section

Figure 3.7: **Adjustments to a shock to tradable endowment.**



Note: Impulse responses are given in percentage deviations from steady state, except for the interest rate, unemployment, the trade balance and the current account, which are given in absolute deviations.

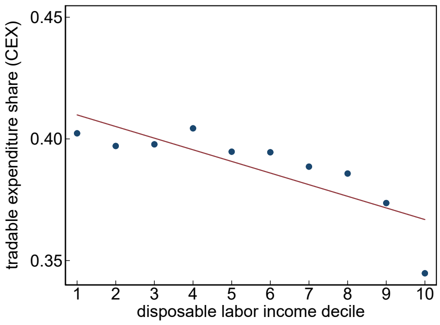
of households with different income and wealth levels. For example, Boppart (2014) uses US survey data to show that poor households spend a relatively larger income share on goods relative to services. Using Mexican micro data, Cravino and Levchenko (2017) compute tradable expenditure shares along the income distribution.[18]

Carroll and Hur (2020) show that tradable expenditure shares are not only negatively correlated with income, but also with wealth holdings. Unfortunately, there are no estimates on heterogeneous tradable shares available for Argentina. In the following analysis, I use Carroll and Hur (2020)'s estimates for the calibration of the non-homothetic utility function, because their estimates are the most recent ones and cover both the income and the wealth dimension. Moreover, their estimated average expenditure share for tradables is similar to Argentina's. The two top panels in figure 3.8 show the tradable expenditure share along the income distribution (left panel) and along the wealth distribution (right panel), as estimated by Carroll and Hur (2020). As indicated by the negative slopes of the red lines, the tradable expenditure share decreases both in income and in wealth. For income, the range is around 0.4 for the bottom decile and less than 0.35 for the top decile. The negative correlation between liquid wealth and the tradable expenditure is even stronger with a share of around 0.44 for the bottom decile, monotonically decreasing to 0.33 for the top decile.

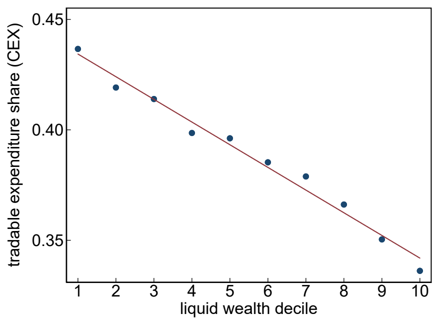
[18]There is also a large empirical literature on the estimation of expenditure elasticities of different product categories. For example, for the US Aguiar and Bils (2015) find that services like food away from home and entertainment fees have the highest expenditure elasticities, and food at home has the lowest.

Figure 3.8: **Heterogeneity in the tradable expenditure share.**

Data



(a) Along the income distribution.

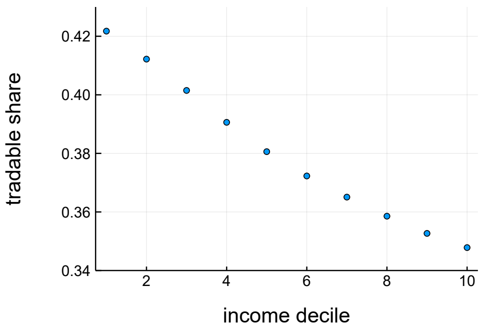


(b) Along the wealth distribution.

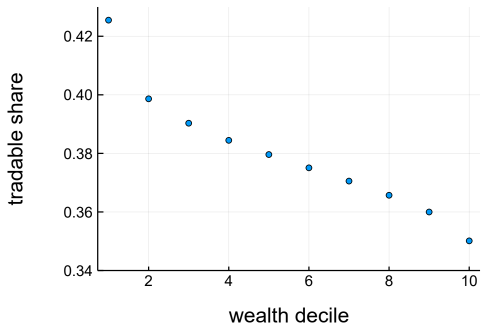
Source: Carroll and Hur (2020).

Note: Wealth includes stocks, real estate, non-corporate business assets, bonds, checking and savings accounts, and vehicles, minus debts. Disposable labor income is computed as total household labor income, the sum of household wages and salaries and 50 percent of farm and business income, plus transfers minus tax liabilities.

Model



(c) Along the income distribution.



(d) Along the wealth distribution.

Note: Tradable shares in the steady state of the model.

Following Carroll and Hur (2020), I specify a Stone Geary utility function of the following form

$$c = \left[\gamma \left(c^T - \bar{c}^T\right)^{1-\frac{1}{\varepsilon}} + (1-\gamma)\left(c^N\right)^{1-\frac{1}{\varepsilon}}\right]^{\frac{1}{1-\frac{1}{\varepsilon}}}, \tag{3.27}$$

where $\bar{c}^T$ depicts the bliss point of tradable consumption. This assumption generates a negative relationship between the income shock and the relative demand for tradables. In combination with the incomplete-markets setup, a modelling choice that is also made by Carroll and Hur (2020), it also produces a negative relationship between wealth and the relative demand for tradables. In the following analysis, $\bar{c}^T$ is set to 0.25 to target the heterogeneity in tradable shares as observed. The resulting tradable shares along the income distribution and along the wealth distribution are depicted in the bottom panels of figure 3.8. Comparison with the two top panels shows that the model is able to reproduce the observed heterogeneity well both for the income and the wealth dimension.

3.4.2 Dynamics with Non-Homothetic Preferences

In the following, I analyze the implications of demand non-homotheticities for the interest rate shock propagation under both exchange rate regimes.

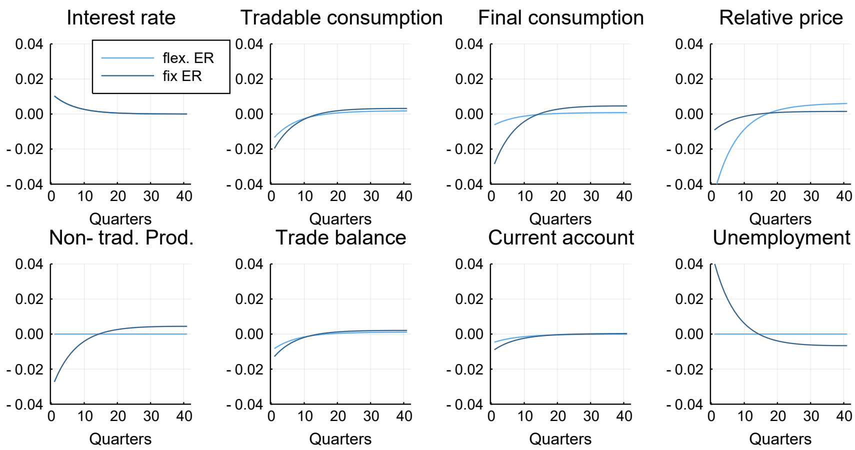
Figure 3.9 shows the dynamics with a flexible exchange rate (light blue) and a fixed exchange rate (dark blue).[19] Comparison with the dynamics under homothetic preferences (see figure 3.5) shows that the presence of non-homothetic preferences amplifies crisis dynamics under both exchange rate regimes, resulting in a sharper devaluation of the real exchange rate in the flexible exchange rate case and in a stronger rise in unemployment with a fixed exchange rate. The reason is that non-homothetic preferences constitute a friction in expenditure switching for poor households. For a given reduction in the relative price, poor households are less willing to switch to the consumption of non-tradable goods. Hence, in the aggregate, the relative price for non-tradable goods has to fall by more to stabilize the domestic economy in the case of a flexible exchange rate. When the exchange rate is fixed, the result is higher unemployment.

Regarding the welfare proxy, the amplification of crisis dynamics brought about by non-homothetic preferences does not carry over into a worsening of the consumption loss under a flexible exchange rate. When the nominal exchange rate can adjust, final consumption drops as much under non-homothetic preferences as under homothetic preferences. However, with a fixed exchange rate, the drop in consumption is considerably stronger under non-homothetic preferences, driven by the adjustment of non-tradables consumption. This is because unemployment surges as a consequence of the inability of the relative price to adjust and, in turn, to incentivize consumption of non-tradables.

In order to analyze the differential impact on households, figure 3.10 shows again the impact responses of final consumption along the income distribution. As with homothetic preferences, poor households suffer the most due to the anti-poor redistributive consequences of the shock. This redistribution, measured by the distance in the consumption adjustment between the top and bottom percentile,

[19] The real exchange rate and the price of the final good cannot be computed when preferences are non-homothetic. For details, see the appendix A.2.1.

Figure 3.9: **Adjustments to an interest rate shock with non-homothetic preferences.**



Note: Impulse responses are given in percentage deviations from steady state, except for the interest rate, unemployment, the trade balance and the current account, which are given in absolute deviations.

is strengthened by non-homothetic preferences. It then amounts to roughly 3% for a flexible and a fixed exchange rate, while it is about 2% and 1.8% respectively under homothetic preferences (see figure 3.6). The explanation depends on the exchange rate regime. With a flexible exchange rate, the relative price declines by so much that even poor households engage in some consumption switching, so their final consumption reduction is only slightly greater than under homothetic preferences. Rich households, whose preferences exhibit no or very little non-homotheticities, benefit from the stronger downward adjustment of the relative price, which makes final consumption more affordable to them. If the exchange rate is fixed, the inability for the relative price to change enough to clear the labor market hurts poor households more than their rich counterparts. Since the former exhibit a strong preference for tradable goods, they keep tradable consumption almost constant but instead reduce non-tradable consumption sharply, reflected in a much stronger drop in final consumption relative to the homothetic case. The impact responses of richer households, in contrast, are almost the same as under homothetic preferences. Since their preferences are (almost) homothetic, so the non-adjusting relative price does not hurt them.

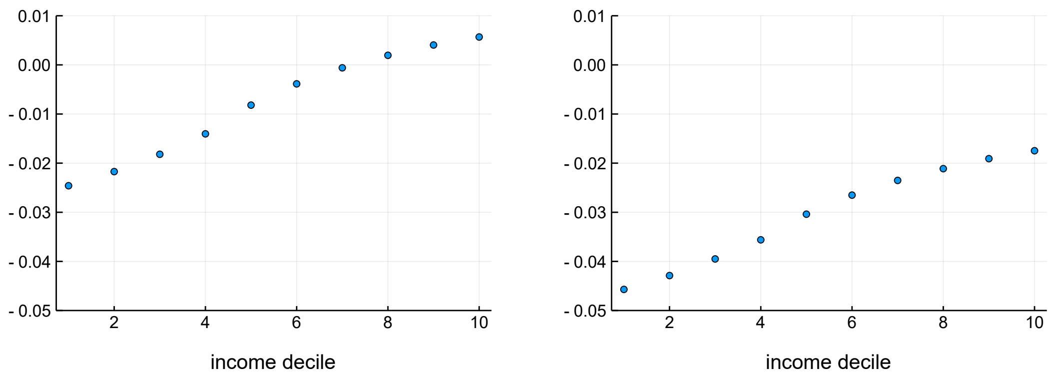
3.4.3 Countercyclical Tax

In the following, I investigate whether a countercyclical tax can act as an automatic stabilizer by stabilizing households' income and thereby mitigate the expenditure-switching friction. To this end, a tax on tradable endowment τ_t, functioning as a countercyclical lump-sum payment, is introduced. The budget constrained then reads as follows

$$c_t^T + p_t c_t^N + a_{t+1} = (1-\tau_t) y_t^T + a_t(1+r_t) + s\frac{W}{\mathcal{E}_t} h_t + \frac{\phi_t}{\mathcal{E}_t}. \tag{3.28}$$

Figure 3.10: **Impact responses of consumption under non-homothetic preferences along the income distribution.**

(a) Flexible exchange rate. (b) fixed exchange rate.



Note: Impact responses are given in percentage deviations from steady state.

To introduce countercyclicality of the income tax, τ_t is modeled as a function of the interest-rate shock process

$$\tau_t = \bar{\tau} - r_t,$$

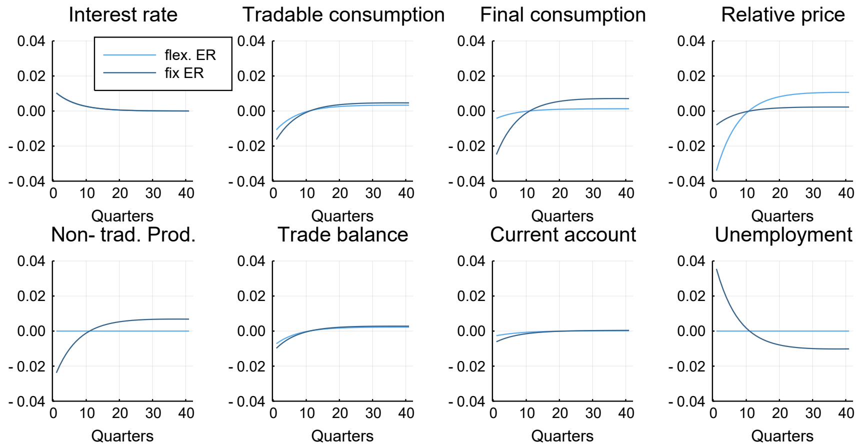
where $\bar{\tau}$ is set to 0.1. Thus, a decline in the interest rate by e.g. 1 percentage point causes a moderate reduction in the tax from 10% to 9%.

Figure 3.11 again shows the economy's adjustment to a shock to the interest rate under both exchange rate regimes. Comparison with figure 3.9 shows that crisis dynamics are dampened for both regimes. This is due to two factors. First, additional income implies that the negative impact on consumption is dampened, which in particular increases consumption of poor, high-MPC households. Second, it neutralizes some of the preference non-homotheticity of poor households, which mitigates the expenditure-switching friction.

3.5 Conclusion

This paper studies the role of household heterogeneity in the transmission of adverse external shocks. To this end, I develop the heterogeneous-household counterpart to SGU (2016)'s RA-SOE model, which features tradable and non-tradable goods, a single asset denominated in foreign currency, and nominal rigidities in the form of DNWR. In my HA version, households are subjected to idiosyncratic shocks and face a borrowing constraint (as in Bewley, 1977), which induces heterogeneity in foreign asset holding. I find that the adverse shock mainly transmits via negative indirect effects on households' real income, whereby poor households are hit harder under both considered exchange rate regimes. Comparison of the adjustment under the fixed and the flexible exchange rate system shows that high-income consumers benefit dis-proportionally from the stabilization provided by an exchange rate devaluation.

Figure 3.11: **Adjustments to an interest rate shock with a counter-cyclical tax.**



Note: Impulse responses are given in percentage deviations from steady state, except for the interest rate, unemployment, the trade balance and the current account, which are given in absolute deviations.

In a second step, I extend the model to include non-homothetic preferences calibrated to match heterogeneous tradable expenditure shares both along the income and the wealth distribution. I find that this generates a friction preventing poorer households from switching from tradable to non-tradable consumption, which amplifies aggregate crisis dynamics, reflected in a stronger devaluation of the real exchange rate (with a flexible nominal exchange rate) or higher unemployment (with a fixed exchange rate). Final consumption drops by more with a fixed exchange rate, relative to the scenario with homothetic preferences, which is driven by poor households not being able to substitute away from tradable towards non-tradable consumption.

It is shown that an exchange rate policy aimed at clearing the labor market replicates the flexible-wage outcome, since a reduction in the nominal wage and a devaluation of the nominal exchange rate bring about the same income effect. The reason is that, since all debt is denominated in foreign currency, it is only repayment capacity measured in real terms that matters. This result is likely to change when only a part of households' asset holdings is denominated in foreign currency and a nominal devaluation thus brings about heterogeneous revaluation effects (see, e.g., De Ferra et al., 2020). The introduction of a second asset denominated in domestic currency is thus a promising avenue of future research.

Chapter 4

Preparing for Longer Lives with Age-Dependent Wage Uncertainty

Joint work with Philipp Engler

Abstract

We analyze how age-dependent wage volatility shapes the adjustment of labor supply and savings to an increase in life expectancy, both over the individual life cycle and at the aggregate level. To this end, the wage process in an otherwise standard overlapping-generations Aiyagari model is modified to generate the empirically observed u-shaped pattern of wage volatility over the life cycle. Relative to the standard model with age-independent wage volatility, we find a more significant role of labor supply in insuring against a higher life expectancy, while precautionary savings are less relevant. In the aggregate, these individual life-cycle patterns dampen the impact of aging on the natural interest rate: Based on the 2000-2050 UN demographics forecast for the US, our model predicts a decline in the natural rate that is ten basis points smaller than in the standard model. Moreover, raising the retirement age to limit the decrease of the natural rate proves less effective.

JEL classification: D15, D31, E21, E24, J22.
Keywords: Earnings risk, savings, labor supply, inequality, life cycle.

4.1 Introduction

Demographic change has been a global phenomenon over the last decades and is considered a key cause of the so-called "savings glut" (Bernanke, 2005).[1] While the original idea refers to a decline in population growth, rising life expectancy is another prominent driver of aging societies. Theoretically, an increase in life expectancy has an ambiguous effect on aggregate savings: Workers accumulate more savings at every age to fund a longer retirement spell ("life cycle effect"), while at the same time the population share of the elderly, who tend to dis-save, increases ("composition effect").[2] Which of the two effects dominates depends on the characteristics of the demographic transition and on the retirement system in place. Bloom et al. (2003, 2007) show that rising life expectancy, which outpaces increases in the retirement age in many countries, is an important driver of increased savings rates – indicating dominance of the life cycle effect.[3] One potential determinant of savings that has not been studied in this context is the empirically documented variability of wage volatility over the life cycle.

This paper analyzes how age-dependent wage volatility shapes the adjustment of labor supply and savings to an increase in life expectancy, both over the individual life cycle and at the aggregate level. To this end, we make two modifications to an otherwise standard overlapping-generations (OLG) Aiyagari (1994) model. First, there is endogenous labor supply (as in Conesa et al., 1999),[4] which has been shown to have a precautionary role that is at least as important as the accumulation of savings.[5] Second, the wage process is modified to generate a u-shaped pattern of wage volatility ("wage risk") over the life cycle, as documented by, e.g., Karahan and Ozkan (2013).[6] We focus on the implications of these life-cycle variations in wage risk, which have not been accounted for in existing models that assume constant wage risk.[7]

We first analyze the implications of age-dependent wage risk for a given demographic structure by examining the initial steady state (prevailing before the model is subjected to an exogenous increase in life expectancy). It is compared across two model versions: One where wage volatility matches its empirically observed u-shaped life-cycle profile (our benchmark model) and one with age-invariant wage risk (the canonical model). For a meaningful comparison, both versions have the same total wage risk, understood as the average wage volatility over the life cycle.[8] Moving from the canonical to the

[1]Other potential explanations that have been brought forward are reduced investment opportunities in industrial countries, depressed technological progress, and a surge in demand for reserves in emerging economies.

[2]Bequest motives reduce dis-saving, but in the aggregate elderly are shown to deplete at least a part of their wealth (see, e.g., Weil, 1994).

[3]In many advanced economies, pension entry age is constant or declining. Exceptions are Germany, the UK and the US. In the US, however, the full retirement age has increased only from 65 to 66, going up to 67 over the next several years, while life expectancy will grow by 6.5 years between 2000 and 2050, according to UN forecasts.

[4]As there are no adjustment costs, I will use "endogenous" and "flexible" labor supply interchangeably in the rest of the paper.

[5]Precautionary behavior of labor supply refers to an increase in labor supply in response to an increase in uncertainty. Adjustments in labor supply provide insurance against both idiosyncratic wage shocks (see Pijoan-Mas (2006) and Low, 2005) and increases in life expectancy (see He et al., 2019). Intuitively, the ability to adjust labor supply in response to shocks reduces the need to self-insure through savings.

[6]For further references, see the literature review at the end of the introduction.

[7]For an overview, see De Nardi and Fella (2017).

[8]We also equalize the total wage risk in all other model comparisons in the paper.

benchmark model tilts the life-cycle pattern of savings towards younger ages, and increases labor supply at young and old ages relative to middle ages. Given the demographic structure in the initial steady state, this implies higher equilibrium capital supply and a lower labor supply at the aggregate level.

The question how age-varying wage risk affects the adjustment to demographic changes is answered in four experiments. The first experiment compares in both models the initial steady state (calibrated to the US in the year 2000) with a terminal steady state in which life expectancy equals its forecasted value for 2050.[9] In our benchmark model, a higher life expectancy increases savings at all ages, while hours adjust only mildly, translating into a drop of the equilibrium real interest rate from 6.00% to 5.547%. The role of age-variant wage risk is identified by comparing these results with those obtained under the canonical model for the same increase in life expectancy. Age-variant wage risk causes the accumulation of additional savings to occur at a younger age, but only has minor implications at the aggregate level: The real interest rate decline is 3 basis points smaller than under constant wage risk.

The second experiment investigates the role of endogenous labor supply in adjusting to a higher expected life expectancy. To this end, we fix labor supply at its initial steady state level, thereby ruling out the adjustment of working hours. Aggregate savings in our benchmark model increase significantly more when hours are fixed, in line with He et al. (2019). The inability to adjust hours increases the need to accumulate more savings in response to a longer expected time in retirement, because hours cannot be used as an adjustment margin to wage shocks.[10] In the aggregate, the greater need to hike savings under fixed labor supply amplifies the drop in the real interest rate, which declines by 0.558 ppts instead of 0.453 ppts under endogenous labor. A comparison to the canonical model shows that age-dependent wage risk reduces the extent to which savings are increased to self-insure for a longer expected retirement spell, associated with a 8-basis-points smaller drop of the natural interest rate. Thus, the importance of elastic labor supply as potential adjustment margin to an increase in life expectancy is particularly important when wage risk is age-dependent.

The third experiment analyzes a more encompassing and realistic scenario of demographic change with both higher life expectancy and a reduced population growth rate from 1.1% in 2000 to 0.5% in 2050, as projected by the UN. In the benchmark model the interest rate declines by 79 basis points, well in line with other estimates by, e.g., Krueger and Ludwig (2007), who predict a 86-basis-point drop for the US between 2005 and 2080. Age dependency of wage risk makes a significant difference for aggregate outcomes: In the benchmark model the demographic transition increases precautionary savings by less than in the canonical model with constant wage risk, reducing the magnitude of the drop in the real interest rate by 10 basis points.

The fourth experiment extends the previous one by also raising the retirement age in the terminal steady state by four years, which stabilizes the old-age dependency ratio for the forecasted life expectancy and population growth rate. An increase in the retirement age reduces the need for precautionary savings, which makes it an effective tool in cushioning the fall in the equilibrium interest rate. However, under age-dependent wage risk, the effectiveness is limited by the fact that the wage earned

[9] We use the forecast by the US Social Security Administration, for details see subsection 4.2.6.

[10] For example, a negative wage realization at the end of the working life poses a greater risk for the financing of the retirement period (and thereby a greater need for precautionary savings) when it cannot be balanced by raising hours.

during the additional employment period is very volatile, which leads households to reduce precautionary savings by less. As a result, raising the retirement age dampens the drop in the natural interest rate by only 39 basis points, compared to almost 46 basis points in the canonical model.

This paper is related to several strands of literature. The analysis of consumption and savings over the life cycle goes back to simple life-cycle OLG models as in Blanchard (1985). Here, young households save for their future retirement and retired households live off their previously acquired wealth. In absence of changes in the population structure or bequest motives, saving and dis-saving cancel out, but positive savings rates can be explained when accounting for positive long-term growth in output and population. However, this framework fails to explain very high national savings rates, as observed for example in China and other parts of Asia. Bloom et al. (2003, 2007) show that increasing life expectancy can explain part of this surge in savings rates. While the aforementioned papers consider several representative cohorts, our paper employs a Bewley (1977) OLG model, in which also households within one cohort are heterogeneous as they are hit by idiosyncratic shocks. This feature is important to capture distributional implications on macroeconomic outcomes and vice versa. The methodology of this paper is shared by several other articles using Bewley (1977) life-cycle models. An overview can be found in De Nardi and Fella (2017). Our model follows Conesa et al. (1999) who allow for flexible labor supply. Takahashi (2019) is, to the best of our knowledge, the only other paper that embeds age-dependent idiosyncratic risk in a general-equilibrium framework, but he studies the implications of age-variant risk for business cycle implications.

Our paper is in the tradition of several contributions that analyze the effects of demographic change on the real interest rate using medium-scale OLG models with idiosyncratic risk. One can broadly distinguish between closed-economy models (see, e.g., Gagnon et al. (2016) and Carvalho et al. (2016) for the US and, e.g., Papetti (2019) for Europe) and multi-country models (see, e.g., Krueger and Ludwig, 2007). They all conclude that demographic change is an important driver of lower equilibrium interest rates. For example, Papetti (2019) attributes a decline of 0.4 ppts between 2000 and 2030 to increased life expectancy. Recently, He et al. (2019) show in a similar model that an increase in life expectancy and a reform of the pension system both contributed to high national savings rates in China.

We use estimates from the empirical literature on wage and labor earnings processes.[11] Traditionally, these processes have been estimated with the assumption of a constant conditional variance, restricting uncertainty to stay constant along the life cycle. Karahan and Ozkan (2013) have shown that the variance of the wage process is not constant along the life cycle, but displays a u-shaped profile. Their findings have been corroborated by several other studies, see, e.g., Guvenen et al. (2014), Blundell et al. (2015), De Nardi et al. (2020), and Sanchez and Wellschmied (2020). Possible explanations are many job-to-job transitions at young ages (see Topel and Ward, 1992) and an increasing risk of health shocks for elderly workers (see Hosseini et al., 2018).

Several papers point to labor supply as an important self-insurance channel, both against wage shocks and against increased life expectancy. Low (2005) uses a simulated life-cycle model to show that

[11] Most empirical studies use earnings data, which is the product of hours and wage. With this dependent variable, a u-shaped variance profile cannot be directly be attributed to wage risk, as it could also result from labor supply adjustments. However, Karahan and Ozkan (2013) document a u-shaped variance profile using wage data.

wage uncertainty induces young households to work more hours relative to their elder counterparts, for whom much of the uncertainty has resolved.[12] In Pijoan-Mas (2006)'s general-equilibrium model, households extensively use the labor supply channel to self-insure against wage risk. He et al. (2019) show that adjustments in labor supply also provide insurance against an increase in life expectancy. All of these papers assume that the wage process has a constant variance over the life cycle.

The inclusion of age-varying wage risk poses computational challenges, as it prevents the use of standard methods to discretize the continuous idiosyncratic process by a Markov Chain. We use Fella et al. (2019)'s non-stationary extension of the Rouwenhorst (1995) method. Some earlier contributions have also used less systematic approaches to generate age-variant transition matrices, among them Kaplan (2012) and Karahan and Ozkan (2013).

The remainder of this paper is organized as follows. In the next section, we set up the theoretical model and present our calibration strategy. In section 4.3, we compare the initial steady states under age-invariant and age-dependent wage risk. Section 4.4 analyzes at the terminal steady state for a constant population growth rate. In section 4.5, we consider broader demographic change, reflecting increased life expectancy and a lower population growth rate. We conclude in section 4.6.

4.2 Model

The model is a standard Aiyagari (1994) OLG model with endogenous labor supply à la Conesa et al. (1999), which is augmented by an age-dependent conditional variance of the idiosyncratic wage process.

4.2.1 Demographics

Time is discrete and in each period j, a continuum of ex ante identical agents is born. An individual lives for a maximum of J periods, so the economy is populated by J overlapping cohorts. The first period of an agent's life corresponds to the beginning of its working life. In period J_R, the agent retires. In each period, there is an age-dependent, positive probability of dying, conditional on being alive at age $j-1$, and the conditional survival probability at age j is denoted by s_j. The unconditional probability to reach age j is thus $\prod_{i=1}^{j} s_i$. At age J, death is certain. The population grows at an exogenous rate n. The size of the total population is normalized to one and the constant measure of cohort j is denoted by μ_j. At any point in time, the fraction of individuals of age j is then given by $\mu_{j+1} = \frac{s_{j+1}}{(1+n)}\mu_j$.

4.2.2 Preferences and Endowment

At birth, each individual maximizes expected lifetime utility, which is given by

$$\mathbb{E}_0 \sum_{j=1}^{J} \beta^{j-1} \left(\prod_{t=1}^{j} s_t \right) U(c_j, n_j), \tag{4.1}$$

[12] Flodén (2006) confirms Low's findings in an analytical paper.

where c_j and n_j denote consumption and hours worked at age j, respectively, and $0 < \beta < 1$ is the discount factor. Agents are endowed with one unit of time to be allocated between leisure and labor. The wage on the competitive labor market is denoted by w, which is taken as given, and households pay a constant social security contribution rate θ. Individual labor productivity e_j is composed of a deterministic age-dependent component ϕ_j and an idiosyncratic shock z_j, so $e_j = \phi_j\, z_j$. The idiosyncratic shock process is given by an AR(1) process with an age-dependent variance $\sigma^2_{\zeta_j}$

$$
\begin{aligned}
z_j &= \rho\, z_{j-1} + \zeta_j, \\
\zeta_j &\overset{iid}{\sim} \mathcal{N}(0,\, \sigma^2_{\zeta_j}).
\end{aligned}
\tag{4.2}
$$

Note that the canonical, variance-stationary process is nested in this formulation for $\sigma^2_{\zeta_j} = \sigma^2_{\zeta} \;\; \forall\ j$. The process is approximated by an age-dependent Markov Chain.

Individuals have access to a one-period non state-contingent bond a to self-insure against idiosyncratic shocks by building up wealth in the spirit of Aiyagari (1994) and other incomplete-market models. Households face the following period budget constraint

$$
c_j + a_{j+1} = (1 + r)\, a_j + (1 - \theta)\, e_j(z_j)\, w\, n_j + p_j, \tag{4.3}
$$

where r is the interest rate and p_j is a social security benefit that is zero before retirement and becomes a positive and constant lump-sum transfer p for all $j \geq J_R$, both r and p_j are taken as given.[13] Labor supply is zero in retirement, i.e., $n_j = 0 \;\; \forall \;\; j \geq J_R$. Furthermore, households face a no-borrowing constraint

$$
a_{j+1} \geq 0. \tag{4.4}
$$

Note that in this environment, there are some accidental bequests, since the savings decision is taken before a potential death occurs. We assume that government consumption is financed through these accidental bequests (see Definition 1 below).

4.2.3 Individual Dynamic Programming Problem

The individual's optimization problem can be formulated as an age-dependent dynamic programming problem. An agent of age $j < J_R$ solves the following problem

$$
\begin{aligned}
V_j(a_j, z_j) = \max_{c_j, n_j, a_{j+1}} \; & U(c_j, n_j) + \beta s_{j+1} E_j V_{j+1}(a_{j+1}, z_{j+1}) \\
\text{s.t.} \quad & c_j + a_{j+1} = (1 + r)\, a_j + (1 - \theta)\, e_j(z_j)\, w\, n_j, \\
& c_j \geq 0, \quad a_{j+1} \geq 0, \quad 0 \leq n_j \leq 1 \quad a_0 = 0.
\end{aligned}
\tag{4.5}
$$

Note that it is assumed that all agents are born without wealth. For a retired individual (i.e., for all $J \geq j \geq J_R$), the dynamic programming problem is as follows

$$
V_j(a_j) = \max_{c_j, a_{j+1}} U(c_j, n_j) + \beta s_{j+1} E_j V_{j+1}(a_{j+1}) \tag{4.6}
$$

[13]Note that relative prices w and r, and the social security contribution rate θ do not have a time index, since I consider an economy in its steady state.

$$\text{s.t.} \quad c_j + a_{j+1} = (1+r)\, a_j + p$$
$$n_j = 0, \quad c_j \geq 0, \quad a_{j+1} \geq 0.$$

Note that in optimum it has to hold that $a_J = 0$, so we can solve for the policy functions by backward induction.

4.2.4 Law of Motion of the Distribution

The Markov Chain and the policy functions induce the following recursive equation for the distribution of individual states $F(a_j, z_j)$

$$F_{j+1}(a_{j+1}, z_{j+1}) = \sum_{z_j} \pi(z_{j+1}|z_j) F_j((a_{j+1}^{-1})(a_{j+1}, z_j), z_j), \tag{4.7}$$

where $\pi(z_{j+1}|z_j)$ denotes the exogenously given transition probability from productivity state z_j to z_{j+1} and $(a_{j+1}^{-1})(a_{j+1}, z_j)$ denotes the inverse of the function for the optimal next-period capital stock $a_{j+1}(a_j, z_j)$ with respect to its first argument a_j.

4.2.5 Equilibrium

There is a representative firm that is owned by households and that produces one good with production technology $Y = F(K, L)$, where K and L denote aggregate capital and labor, respectively. Capital depreciates at rate δ. The labor and capital markets are perfectly competitive, so $w = F_L(K, L)$ and $r = F_K(K, L) - \delta$. There is no role for the government except financing government consumption G from accidental bequests and collecting social security contributions to pay lump-sum pensions to retirees. We will consider a steady state where factor prices and aggregate capital and labor are constant, and the distribution of wealth is stationary.[14]

Definition 1: *A stationary equilibrium is given by* $\{c(x,j), g(x,j), l(x,j), r, w, K, L, G, \theta, p\}$ *and distributions* $\{F_1, F_2, \cdots, F_J\}$ *such that:*

1. *Given relative prices* $\{w, r\}$ *and government policies* p *and* θ*, the individual policy rules* a_{j+1}*,* c_j *and* n_j *solve the household's problem.*
2. *The distributions are consistent with individual behavior.*
3. *Relative prices* $\{w, r\}$ *solve the firm's profit optimization problem.*
4. *Markets clear:*
   (a) *Goods:* $\sum_j \sum_a \sum_z \mu_j \left[c_j(a_j, z_j)\right] F_j(a_j, z_j) = F(K, L) - \delta K - G.$
   (b) *Capital:* $K' = \sum_j \sum_a \sum_z \mu_j \left[a_{j+1}(a_j, z_j)\right] F_j(a_j, z_j).$
   (c) *Labor:* $\sum_{j=1}^{J_R} \sum_a \sum_z \mu_j \left[n_j(a_j, z_j) e_j(z_j)\right] F_j(a_j, z_j) = L.$

[14] Steady states are computed by using backward induction and finding the aggregate capital stock K and hours worked H by bisection. For details, see appendix A.3.1. For more details on the equilibrium concept, see Heer and Maussner (2009).

5. *Social security feasibility:* $\theta w L = p\Big[\sum_{j=J_R}^{J} \mu_j\Big]$.
6. *Government consumption is exogenous and financed by accidental bequests:*
$G = \Big[\sum_{t=1}^{J-1} \frac{\mu_t\,(1-s_{t+1})}{1+n}\Big] \sum_a \sum_z \big[a_{j+1}(a_j, z_j)(1+r)\big] F_j(a_j, z_j)$.

4.2.6 Functional Forms and Calibration

As our model is a variation of Conesa et al. (1999) and our starting point is also the US economy in the year 2000, we follow their calibration, with two exceptions.[15] First, we use more recent microeconometric evidence for the calibration of the idiosyncratic shock process. Second, as this deviation implies a different capital-output ratio, we calibrate the discount factor to target the original capital-output ratio. Our calibration is summarized in Table 4.1.

Demographics

Individuals enter the labor market at age $j = 1$, which corresponds to age 25 in real life, retire at age $J_R = 41$ (corresponds to real age 65) and face a certain death at $J = 61$ (corresponds to real age 85). The population growth rate is 1.1 % annually, which is the US long-term average. Conditional survival probabilities are computed using life tables provided by the US Social Security Administration.[16] Figure 4.1 shows the implied age distribution of the model, which matches the US Census data well. In line with Conesa et al. (1999), the social security contribution rate is set to 11%, which is chosen to match a replacement ratio of 50%.

Idiosyncratic process

Agents are heterogeneous with respect to their labor productivity that is determined by their age and their realization of the idiosyncratic wage process. The deterministic part is calibrated according to evidence by Hansen (1993) and follows a hump-shaped pattern (see figure 4.2). Hansen (1993) computes the age-productivity profile by dividing the hourly wage of cohort j by the average hourly wage in the whole population.

The stochastic part z follows an AR(1) process. For individual i at age j, it is given by

$$z_j^i = \rho z_{j-1}^i + \zeta_j^i,$$

$$z_1^i \overset{id}{\sim} \mathcal{N}(0,\, \sigma_{z_1}^2)\,,\ \zeta_j^i \overset{iid}{\sim} \mathcal{N}(0,\, \sigma_{\zeta_j}^2)\,,$$

where $\sigma_{\zeta_j}^2$ is the variance of z_j^i conditional on a realization z_{j-1}^i, which can be age-variant. In the following analysis, we compare outcomes under the canonical, variance-stationary specification (i.e., $\sigma_{\zeta_j}^2 = \sigma_\zeta^2 \ \forall\ j$) with a specification where the conditional variance, $\sigma_{\zeta_j}^2$, is age-dependent, using esti-

[15]Their calibration strategy targets long-run averages of the US economy. For details see Conesa et al. (1999).
[16]The source is the following report: *Life Tables for the United States Social Security Area 1900-2100—Actuarial Study No. 120.*

Table 4.1: **Calibration of model parameters.**

Parameter		Value	Target/Source
Age of retirement	J_R	41	Corresponds to real age 65
Age of certain death	J	61	Corresponds to real age 85
Population growth annually	n	1.01	US long-term average
Survival probabilities	$\{s_j\}$		2000 US Census
Social security rate	θ	0.11	Conesa et al. (1999)
Autocorrelation idios. process	ρ	0.9	Avg. value from Karahan and Ozkan (2013)
Cond. variance (age-dep. process)	$\{\sigma^2_{\zeta_j}\}$		Karahan and Ozkan (2013)
Cond. variance (age-inv. process)	σ^2_ζ	0.0235	Same avg. cond. var. as age-dep. process
# states of Markov Chain	N	10	High accuracy of appr. of processes
Cobb-Douglas weighting factor	α	0.36	Conesa et al. (1999)
Depreciation rate	δ	0.08	Conesa et al. (1999)
CRRA	σ	2	Conesa et al. (1999)
Consumption weight	γ	0.42	Conesa et al. (1999)
Discount factor	β	0.9624*	Implies capital-output ratio of 3

*Applies to the initial steady state of the benchmark experiment.

Figure 4.1: **Age structure in the US in 2000: Model vs. US Census data.**

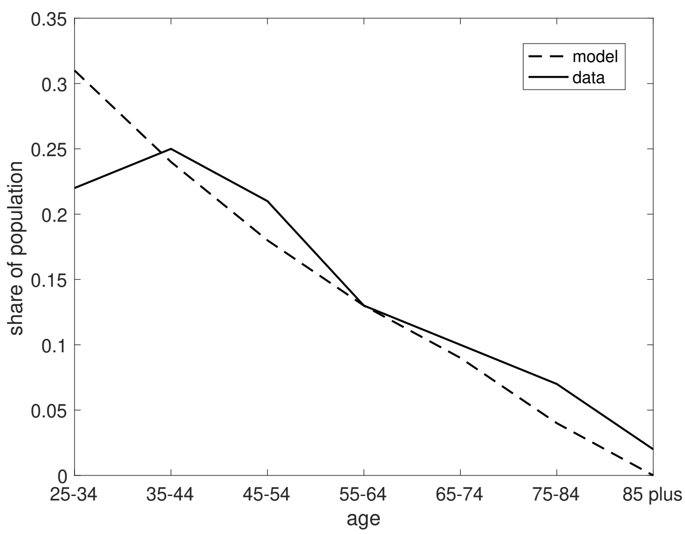
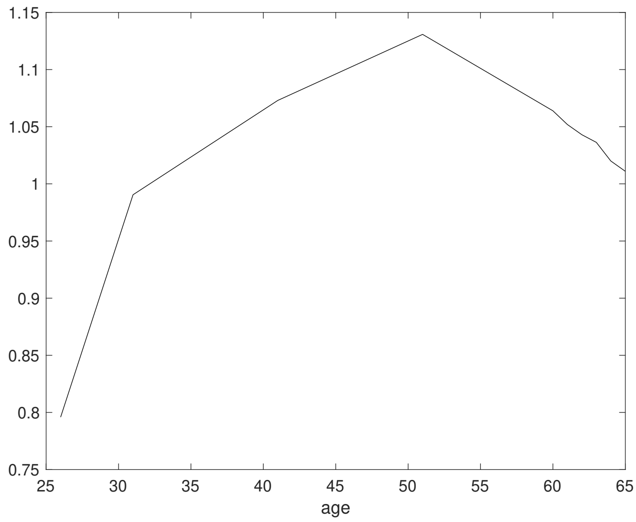


Figure 4.2: **Deterministic efficiency profile.**



Source: Hansen (1993).

mates from Karahan and Ozkan (2013).[17] More specifically, we use their estimation results for a cubic specification of the idiosyncratic earnings process, according to which the conditional variance at age j is governed by

$$\sigma^2_{\zeta_j} = 0.0518 - 0.0405\frac{j}{10} + 0.0105\left(\frac{j}{10}\right)^2 - 0.0002\left(\frac{j}{10}\right)^3.$$

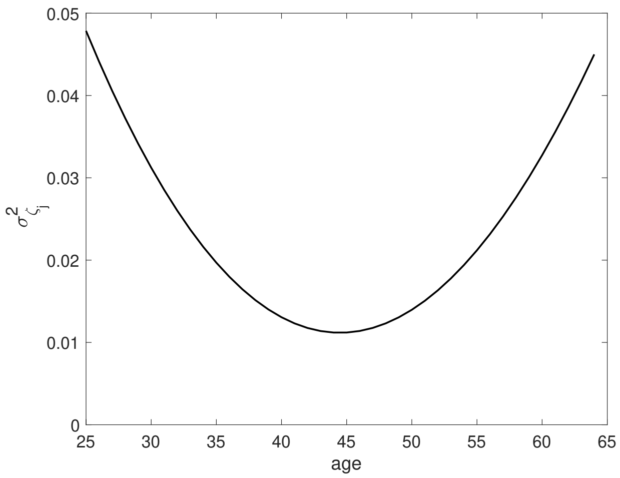
The resulting u-shaped life-time profile is shown in figure 4.3. When we compare the process with age-variant wage risk ("benchmark process") and the process with constant wage risk ("canonical process"), we set the constant conditional variance of the canonical process to 0.0235, which is the average life-time variance of the benchmark process.[18]

The canonical and the benchmark process are approximated using the standard Rouwenhorst (1995) method and Fella et al. (2019)'s non-stationary extension of this method, respectively. The state space is given by $\Upsilon^N = \{\bar{z}^1, ..., \bar{z}^N\}$ in the variance-stationary case and by $\Upsilon^N_j = \{\bar{z}^1_j, ..., \bar{z}^N_j\}$ in the case with an age-dependent variance, where $N = 10$. For the canonical process, the transition matrix Π^N

[17]Ideally, we would have estimates for the wage, and not the earnings process, because earnings can change for a given wage when hours change. In the appendix, Karahan and Ozkan (2013) also perform a robustness exercise with wage data, but only for three age bins. The estimates for the age bins are very similar for the wage and the earnings data, pointing to a similar age profile of the earnings and the wage process. In the Working Paper version, they also test a cubic specification for wage data and it turns out that both processes have very similar properties.

[18]The estimate of the conditional variance in the variance-stationary specification in Karahan and Ozkan (2013) is lower than the average conditional variance of the process with age-dependent wage risk. Using their estimate would imply an unfair comparison, as the first-order effects of a lower average life-cycle risk would dominate the second-order effects implied by a difference in the variation of the risk over the life cycle.

Figure 4.3: **Life-cycle profile of conditional variance.**



Source: Karahan and Ozkan (2013).

with $\Pi_{zz'} = Prob\,(z_{t+1} = z' | \; z_t = z)$ is age-invariant, while for the benchmark process the transition matrix Π_j^N and related transition probabilities $\Pi_{zz',j} = Prob_j\,(z_{t+1} = z' | \; z_t = z)$ are age-dependent.

Preferences and technology

Functional forms and calibration of household preferences and production technology follow Conesa et al. (1999), with the exception of the discount factor. The production function has a standard Cobb-Douglas form

$$F(K, L) = K^\alpha L^{1-\alpha},$$

where parameter $\alpha = 0.36$ is chosen to match the labor share of output. The annual depreciation rate δ is set to 6%. Preferences over consumption and leisure are represented by a utility function of the form

$$U(c, n) = \frac{1}{1-\sigma}(c^\gamma n^{1-\gamma})^{1-\sigma},$$

where relative risk aversion $\sigma = 2$ and consumption weight $\gamma = 0.42$. Finally, discount factor β is calibrated to target a capital-output share of 3. Note that the resulting discount factor $\beta = 0.9624$ is lower than 0.97 as in Conesa et al. (1999). The difference results from the higher average life-cycle risk implied by Karahan and Ozkan (2013)'s estimates, compared to the specification used by Conesa et al. (1999). To target the same capital-output ratio, we assume a lower discount factor.[19]

[19] An alternative calibration strategy would be to match all three "deep" household parameters, σ, γ, and β, simultaneously to match the three long-run values targeted by Conesa et al. (1999). As we, however, need to re-calibrate the model for different specifications, the resulting differences in the three parameters would make it difficult to compare and interpret different outcomes under the two different processes.

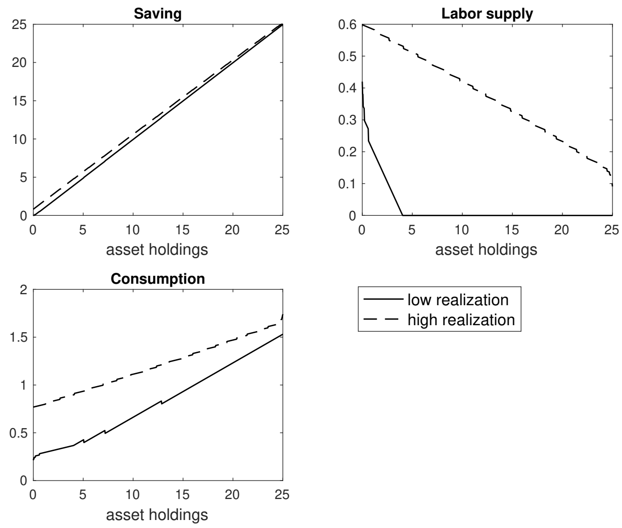
4.3 Age-Invariant vs. Age-Dependent Idiosyncratic Risk

This section analyzes the implications of age-variant wage risk for a constant demographic structure, i.e., before changes to life expectancy or the population growth rate. This is useful to isolate the differences that arise only in response to the change in the demographics.

4.3.1 Life Cycle Simulations at the Individual Level

Before turning to the general equilibrium in subsection 4.3.2, we analyze differences in individual life-cycle behavior under the two different wage processes in partial equilibrium, i.e., for a fixed interest and wage rate.[20] This is instructive for two reasons. First, a realistic life-cycle profile of dispersion supports the adequacy of the corresponding model specification. Second, differences in mean behavior are crucial to understand how age-dependent wage risk changes the asset holdings and labor supply of the individual household. Second,

Figure 4.4: **Policy functions of a working agent at age 40.**



Note: The solid line refers to a worker with the lowest possible idiosyncratic wage realization (i.e., the lowest possible state of the Markov Chain), the dashed line refers to a worker with the highest possible realization (i.e., the highest possible state of the Markov Chain).

First we analyse in a simple life-cycle model how hours and asset holdings over the life cycle depend on realizations of labor productivity. Figure 4.4 shows the policy functions of a working agent at age 40 in the benchmark model for the highest possible (dashed line) and the lowest possible (solid line)

[20]Note that they are fixed at their initial steady-state values to make the comparison with the general-equilibrium results in the next subsection more transparent.

realization of the idiosyncratic productivity shock. Policy functions of agents with intermediate wage realizations lie between these two extremes. For the low-wage agent, the optimal choice for next period's asset holdings as a function of current asset holdings closely resembles a 45-degree line, indicating that current holdings of the one-period asset are rolled over into the next period (when the agent turns 41). That is, net savings are close to zero. Optimal next period's asset holdings are higher for the high-wage worker, especially when this period's asset holdings are low. This shows that these agents can afford to increase their precautionary savings, which is especially important when current asset holdings are low. For both, optimal savings are an increasing function of asset holdings. Optimal labor supply implies that higher asset holdings are associated with both a lower labor supply and a higher optimal consumption level, as richer households can afford to consume more and enjoy more leisure. The comparison between high-wage and low-wage agents shows that the substitution effect of a higher wage dominates the wealth effect: High-realization households supply more labor to afford more consumption.

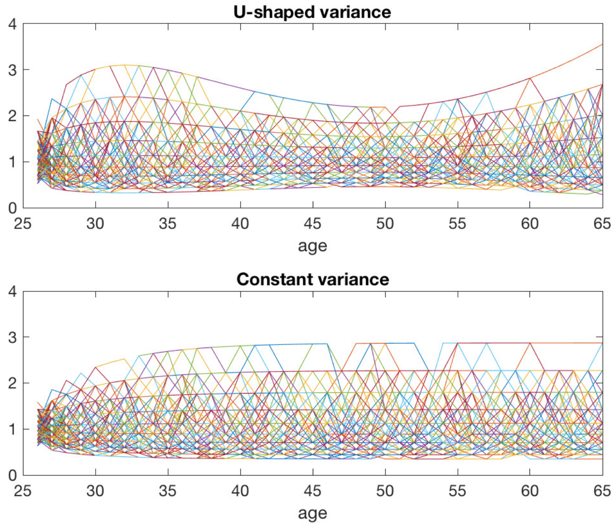
We now examine the two idiosyncratic wage processes in isolation. To that end, we compute individual life cycles for 10000 simulated realizations of the wage processes and study the cross-section for different age groups. Since all agents are born at the same point in time, cross-sectional moments tell us about the distribution of hours and asset holdings resulting from optimal behavior of an individual agent at a given age. The upper panel in figure 4.5 shows 10000 realizations obtained from simulations with an age-dependent conditional variance $\sigma^2_{\zeta_j}$ ("benchmark process") that is part of our benchmark model. The lower panel shows realizations for a constant $\sigma^2_{\zeta_j}$ ("canonical process") as in the canonical model. The canonical process exhibits an unconditional variance that is relatively small at the beginning of the working life and steadily increases up until middle ages from which point on it remains constant. The benchmark process has a higher unconditional variance at the beginning and towards the end of the working life than in the middle, in accordance with empirical evidence.

Combining the simulated realizations of the two wage processes from figure 4.5 with the policy functions illustrated in figure 4.4 allows us to compute the life-time profiles of hours and asset holdings shown in figure 4.6. The upper (lower) row in this figure shows hours worked (asset holdings), once for simulations with the benchmark wage process (first column), and once for the canonical wage process (second column). The corresponding cross-sectional variances for hours and asset holdings are shown in first and second panel of figure 4.7 respectively, with solid lines for the benchmark wage process and dashed lines for the canonical one.[21] For asset holdings we observe a greater dispersion under the benchmark process at younger ages and a smaller dispersion for workers above 54.

Regarding hours, the benchmark process generates interesting differences of the cross-sectional dispersion, relative to the canonical model. Hours dispersion is greater within the group between 25 and 35 than it is across workers between 35 and 45. For the population above 45, the dispersion increases steeply in the age of the subgroup. This contrasts with the canonical process under which the dispersion increases almost monotonically in the age of the subgroup. Introducing age-dependent wage risk makes the dispersion of hours for different age groups empirically more plausible. As noted by Kaplan (2012), standard life cycle Bewley models cannot capture the decrease of the cross-sectional variance for middle-

[21] The results for the canonical process align with Low (2005)'s results for a similar simulation exercise.

Figure 4.5: **Idiosyncratic wage component.**



aged workers that is observed in the data (also documented by Kaplan, 2012). To generate this feature, Kaplan (2012) has to resort to a non-microfounded modelling choice that generates an idiosyncratic wedge in the intratemporal first-order condition. Our simulations show that a realistic dispersion can also be achieved in a standard life cycle model featuring a more realistic idiosyncratic wage process. While generating realistic savings and hours dispersion is not the primary objective of this paper, it does make us confident that the model with a flexible labor supply choice and age-dependent wage risk is suited for our analysis.

Figure 4.8 shows the differences in the cross-sectional mean life-cycle profiles of hours and savings. Under the benchmark process, more assets are held at young ages and less at middle ages, relative to the canonical process. Introducing a u-shaped profile of $\sigma^2_{\zeta_j}$ thus boosts savings of young households, while middle-aged households save relatively less. This finding is particularly interesting considering that Karahan and Ozkan (2013) have not found any significant differences in mean asset holdings in a similar simulation exercise, but without endogenous labor supply. The difference in the timing of the two forms of precautionary behavior over the life cycle can be expected to have implications for the adjustment to an increase in life expectancy, which is analyzed in section 4.4.

The previous finding hints to an important role of labor supply adjustments as insurance mechanism against idiosyncratic shocks. Flodén (2006) shows that labor supply is increased, alongside saving, when there is higher individual wage risk,[22] while Pijoan-Mas (2006) shows that the precautionary role

[22] According to Flodén (2006), flexible labor supply has two effects on the strength of the precautionary motive. First, under flexible labor supply, hours can be chosen in response to a realization of a wage shock. The direction of this adjustment, however, depends on the specific preferences and, as Flodén (2006) points out, "*[...] the exact ways in which agents use variations in labor supply to insure against shocks are diffuse*". A second potential channel is the adjustment of labor supply in response to the presence

Figure 4.6: **Simulated hours and asset holdings.**

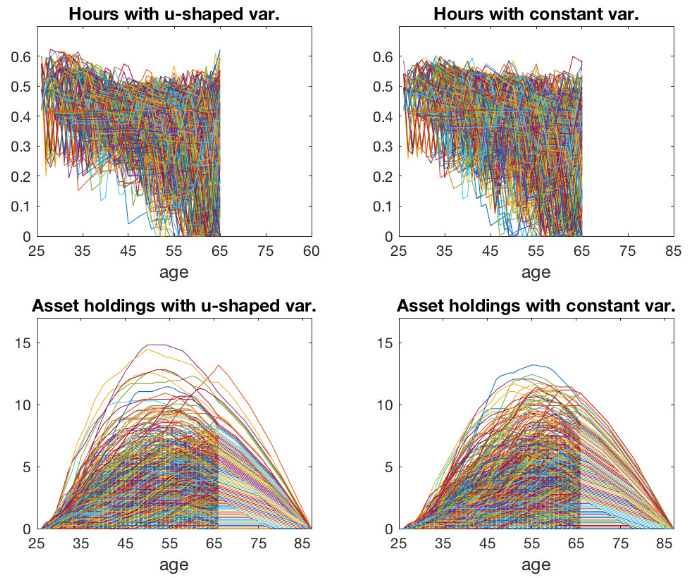


Figure 4.7: **Cross-sectional variance over the life cycle.**

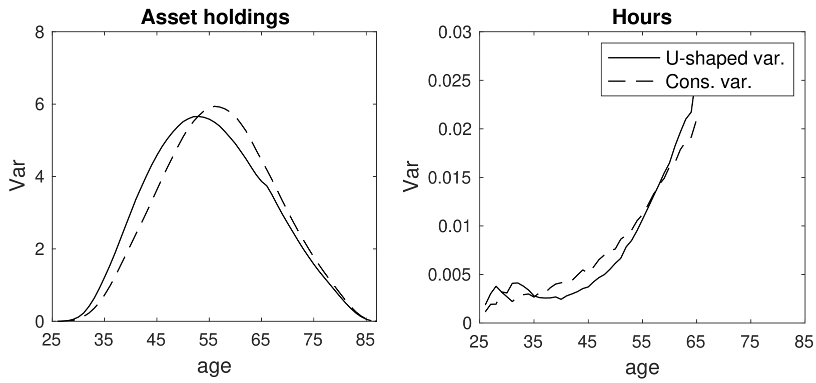
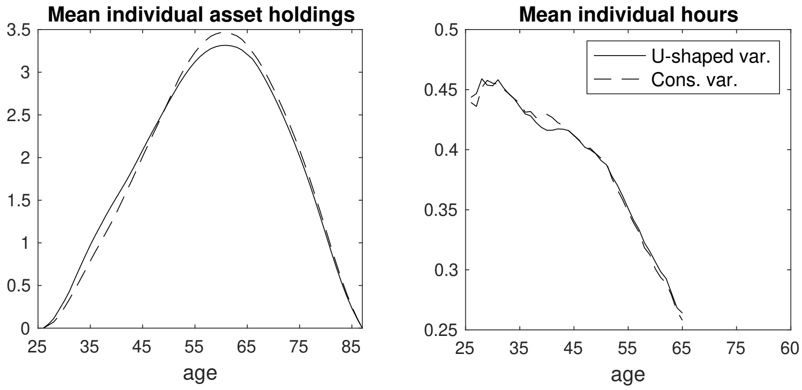


Figure 4.8: **Mean life-cycle profile of hours and asset holdings.**



of labor supply is at least as important as the adjustment of savings. Low (2005) shows that wage uncertainty makes young households work more than elderly, for which much of the uncertainty has already resolved (a pattern that can be detected in our results as well, see figure 4.8).

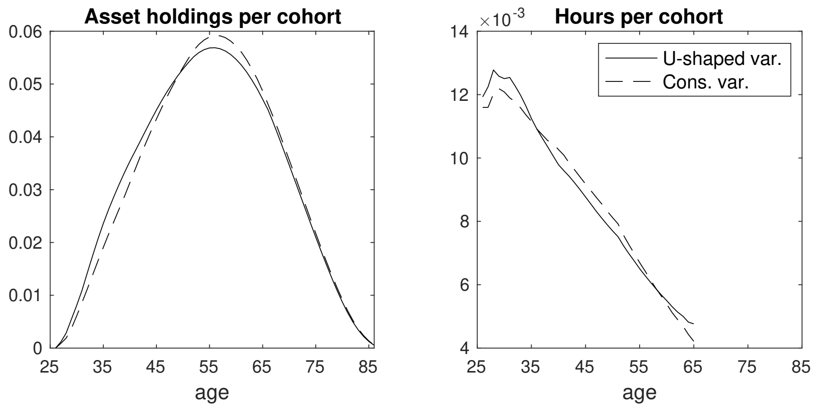
The aforementioned papers only look at variance-stationary processes and, to the best of our knowledge, there is no paper yet that analyzes the life cycle behavior under age-dependent wage risk with endogenous labor supply. Our results show that in the presence of a u-shaped life-time uncertainty profile, households save even more and work more hours when entering the labor market, relative to the canonical process. This might be due to the relatively higher idiosyncratic wage risk at young ages. Middle-aged households supply on average less labor, and thereby gradually reduce their lead in asset holdings relative to their counterparts in a model with constant $\sigma^2_{\zeta_j}$. Note that there is a cut-off point where the difference in relative labor supply reverses: Beginning at age 43, households save less and work slightly more under age-dependent wage risk. Once some of the uncertainty has resolved, households who have build up high precautionary savings at young ages, can afford to save less or to dis-save faster.

4.3.2 General Equilibrium

We now move from comparing the life-cycle behavior of single agents under the two wage processes to comparison of the general-equilibrium implications. The full OLG general-equilibrium model tracks the birth dates of different cohorts and generates the distribution of their population shares (shown in figure 4.1) from a calibrated population growth rate and annual survival probabilities. Furthermore, in the full general-equilibrium model, interest rate, wage, and lump-sum transfers adjust endogenously to ensure market clearing. The stationary general equilibrium derived in this section will be the initial

of uncertainty rather than to specific realization of shocks. To isolate this channel, the author assumes that labor supply can only be chosen after shocks have realized, and finds that households increase labor supply when uncertainty is greater, in order to accumulate more precautionary savings.

Figure 4.9: **Asset holdings and hours per cohort in the initial steady state.**



steady state to which we compare the steady state after a change in demographics.

Figure 4.9 depicts the general-equilibrium distributions of asset holdings (expressed as a share of total assets in the economy) and hours across cohorts, again for the benchmark and canonical wage process (solid and dashed lines, respectively).[23] The general-equilibrium model allows us to study how the differences in the distributions translate into differences in the aggregate capital stock and in aggregate labor supply (both are endogenous and jointly determined with interest rate r, wage w and transfers p). The first two columns in table 4.2 show general-equilibrium values of key variables under the two wage processes. The values in column 1 (benchmark wage process) were computed targeting a capital-output ratio of 3, which gives a β of 0.9624 (as described in table 4.1). The same value of β was used in the computation of the canonical model (column 2). Figure 4.9 shows that introducing age dependency of $\sigma^2_{\zeta_j}$ increases individual asset holdings at the beginning of the working life and reduces them towards its end. A comparison of columns 1 and 2 implies that the additional savings of younger households under the benchmark process exceeds the reduction in savings by older households, leading to an overall higher capital supply. We conclude that not only the total wage risk over the life cycle matters for the degree of precautionary savings, but also how this risk changes over the cycle – i.e., how $\sigma^2_{\zeta_j}$ depends on age. Neglecting the observed u-shaped pattern leads to an underestimation of precautionary savings.

In order to analyze the different implications of the two processes for an increase in life expectancy, the model with the age-invariant process will be re-calibrated with $\beta = 0.9632$ to target the same capital-output ratio of 3 (see column 3).[24] Aggregate hours are negligibly smaller under the age-dependent

[23] The horizontal axis now depicts different cohorts, rather than different ages of the same cohort as in figure 4.8. The resemblance between figures 4.9 and 4.8 owes to the fact that the interest and wage rates in the previous exercise were calibrated to be in line with their general equilibrium values. Differences result from different cohort weights in the general equilibrium.

[24] The reason for re-calibrating the model is to ensure that differences in the adjustment to an increase life expectancy between the two models are driven by the different adjustments to an increased life expectancy only, and not by differences in precautionary savings for a *given* life expectancy.

Table 4.2: **Aggregate values in the initial steady state.**

	Benchmark wage process	**Canonical wage process**	**Canonical wage process (re-calibrated)**
r	6.000%	6.078%	6.000%
w	1.187	1.183	1.187
p	0.236	0.235	0.237
Capital	1.945	1.928	1.952
Hours	0.349	0.350	0.351
Consumption	0.502	0.502	0.502
Output	0.648	0.647	0.651
Capital/Output	3.000*	2.979	3.000*
Welfare	-71.508	-71.326	-71.660
Coefficient of variation	1.046	1.063	1.058

*Calibration target.

$\sigma^2_{\zeta_j}$. When we hold the capital-output ratio constant across the two wage processes (columns 1 and 3), aggregate wage w and pension p are both approximately the same. Aggregate consumption is in all three columns the same, reflecting identical household preferences. Output is slightly higher for the re-calibrated age-invariant specification, mirroring both higher levels of aggregate hours and capital.

We compute aggregate welfare using a consumption equivalent variation measure given by

$$W = \sum_j \sum_a \sum_z \mu_j \left[EV_j(a_j, z_j) \right] F_j(a_j, z_j), \tag{4.8}$$

where the welfare of an individual of type (a, j) is given by

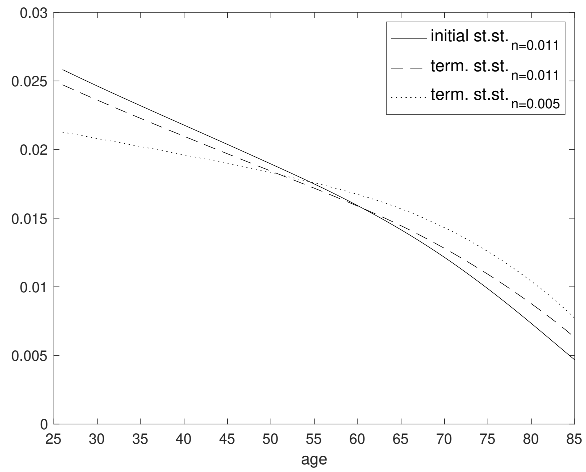
$$EV_j(a_j, z_j) = \left(\frac{V_2(a_j, z_j)}{V_1(a_j, z_j)} \right)^{1-\gamma(1-\sigma)}. \tag{4.9}$$

Welfare is slightly lower under age-dependent wage risk, which is driven by the (additional) need to self-insure against age-variant wage risk. When we re-calibrate the model to hold the capital-labor ratio constant, welfare is slightly lower for the age-invariant process, which is, however, mainly a consequence of the higher discount factor. The coefficient of variation (a measure for inequality) is slightly lower under the age-dependent wage risk, indicating that higher precautionary savings dampen inequality in the model. When we hold constant the capital-output ratio, thereby increase aggregate savings, this difference is reduced.

4.4 Increase in Life Expectancy

This section studies how the adjustment of the model economy to the increase in life expectancy differs between the benchmark and the canonical model, i.e., how it is affected by the u-shaped life-cycle pattern

Figure 4.10: **Composition of the population in the initial and the terminal steady state.**



of wage risk. Taking on a general-equilibrium perspective allows us to shed light on the different long-term implications of increasing life expectancy on the natural interest rate.

4.4.1 Adjustment under Endogenous Labor Supply

We first analyze the economy's adjustment when labor supply is endogenous. For both models we analyze the differences between a steady state with an initial demographic calibration and a terminal steady state in which the demographic calibration has been adjusted. The initial steady state corresponds to the general equilibrium studied in section 4.3.2, in which agents' survival probabilities and the population growth rate were chosen to match the demographics in the US in the year 2000. In the terminal steady state, the survival probabilities are calibrated such that life expectancy equals its UN projection for the year 2050, which is – in the population average – seven years more than in 2000.

Figure 4.10 compares the population shares of different cohorts ("age structure" for short) between the two steady states, with solid (dashed) lines depicting the initial (terminal) one.[25] The population shares in the terminal steady state are the shares that arise when the adjustment to the increase in life expectancy is completed – in other words, shares that have converged to a new stationary distribution. Note that this transition is only completed once the youngest cohort of 2000 has died, i.e., in 2085. Thus, it should not be interpreted as the steady state prevailing in the year 2050. Intuitively, a higher life expectancy reduces the weights of young cohorts (as the overall population size is increased), and increases the weight of older cohorts.

We compare the economy's adjustment when labor supply can be adjusted in response to the increase in life expectancy. Table 4.3 has three columns for each of the two models (with the benchmark

[25]The dotted line shows the terminal distribution under a lower population growth rate, which will be relevant in the next section and can be disregarded for now.

and canonical wage process): Columns titled "2000" and "2050" show key variables in the initial and the terminal steady state, respectively, and columns titled "Δ" compute the difference between the two, i.e., how the respective variable adjusts to the higher life expectancy. Independently of the specific wage process, a higher life expectancy in combination with a constant retirement age lengthens the expected time in retirement and thereby strengthens the incentive to accumulate savings to finance it. Thus, workers save more (or, equivalently, dis-save less) at any given age, which ceteris paribus increases aggregate savings (the life-cycle effect). However, a higher life expectancy also impacts the age structure of the population in a way that ceteris paribus lowers aggregate savings (the composition effect): As apparent in figure 4.10, elderly workers who tend to dis-save become relatively more numerous, which lowers per-capita savings. In our set-up, the life cycle effect dominates the composition effect and the increase in life expectancy brings about additional capital supply, thereby exerting downward pressure on the interest rate. This downward pressure is amplified by the supply side, where the decrease in the share of the working population depresses demand for capital and its marginal product. Both effects jointly decrease the equilibrium interest rate by about 45 basis points. This reduction is slightly stronger in the canonical model, mirroring a greater increase of capital supply. That is, even though capital supply in the initial steady state of the canonical model is lower (before the model is re-calibrated to hold the capital-labor-ratio constant, compare subsection 4.3.2), the increase in savings following a hike in life expectancy is greater.[26] This points to a stronger (weaker) role of precautionary savings for the benchmark (canonical) model in insuring against idiosyncratic wage risk, but to a smaller (greater) importance of precautionary savings to prepare for increased life expectancy. A possible explanation behind this finding is that the possibility to adjust hours is especially important when faced with a higher life expectancy in the presence of age-dependent wage risk. This hypothesis will be further investigated in the next subsection.

In both models, aggregate hours decrease by the same amount as a result of higher life expectancy, and wages decline accordingly. This is driven by the shift of the population structure towards older workers who hold on average more assets and thus have a lower marginal utility of income. Pension benefits p decline as a consequence of the increased old-age dependency ratio, while consumption and output slightly increase. Differences between the two models for other aggregate variables are negligible. There are, however, important differences for welfare and the coefficient of variation between the two models. Higher precautionary savings in the terminal steady state mean lower welfare and a lower coefficient of variation in both models, but the relatively higher additional savings in the canonical model bring about a sharper decrease in welfare and the coefficient of variation is reduced by less.

To understand the drivers of the adjustment of the asset distribution to the rise in life expectancy, we decompose the adjustment into the life-cycle effect, the composition effect, and the general-equilibrium effect. The life-cycle effect is isolated by fixing the interest rate, the aggregate wage and the population structure in the terminal steady state at their values in the initial steady state. This strategy shuts off general equilibrium effects (operating via interest rate and wage) as well as composition effects (operating via the age structure). The joint impact of life-cycle and composition effects is computed by

[26] This also holds when the capital-labor ration is not held constant across the two models by adjusting β.

Table 4.3: **Changes in aggregate variables.**

	Benchmark model			**Canonical model**		
	2000	2050	Δ	2000**	2050	Δ
r	6.000%	5.547%	-0.453%	6.000%	5.543%	-0.457%
w	1.187	1.213	0.026	1.187	1.214	0.027
p	0.236	0.212	-0.024	0.237	0.214	-0.023
Capital	1.945	2.036	0.091	1.952	2.048	0.096
Hours	0.349	0.344	-0.005	0.351	0.346	-0.005
Consumption	0.502	0.506	0.004	0.502	0.506	0.004
Output	0.648	0.653	0.005	0.651	0.657	0.006
Capital/Output	3.000*	3.117	0.117	3.000*	3.118	0.118
Welfare	-71.508	-78.019	-6.511	-71.660	-78.223	-6.563
Coefficient of var.	1.046	1.020	-0.026	1.058	1.045	-0.013

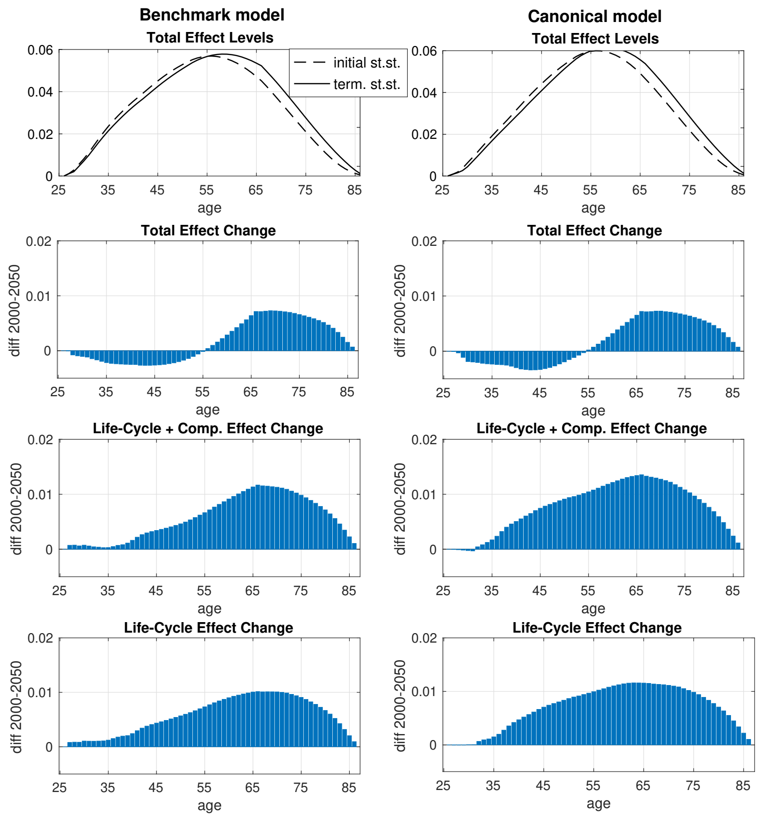
*Calibration target.

**Re-calibrated to target the same capital-output ratio.

allowing the age structure (but not the interest and wage rate) to adjust to the higher longevity in the terminal steady state. The joint impact of all three effects is obtained by allowing all variables to adjust, as done in table 4.2.

Figure 4.11 compares the decomposition between the benchmark and canonical model (first and second column respectively). The first row again shows asset distributions in the initial and terminal steady states and the second row depicts total differences, i.e., the joint impact of all three aforementioned effects. Row three shows the joint impact of only life-cycle and composition effects, while row four shows the life-cycle effect in isolation. Turning to the latter, we observe that the additional savings of younger households (below 35) are greater in the benchmark model, while total additional savings are lower. There are two potential explanations. First, since precautionary savings are generally higher in the benchmark model – especially for younger households, as apparent in figure 4.9 – there is less need to build up further savings in response to increased longevity. Second, the contribution of precautionary labor supply might be larger in this model, a hypothesis that is explored in detail in the following subsection. Now we turn to the joint impact of life cycle and composition effect in the third row. Relative to the bottom panels, additional savings of older cohorts are accentuated relative to those of younger cohorts under both wage processes. Intuitively, the shift of mass in the age distribution towards older households implies that assets per cohort increase the most for the elderly. General-equilibrium effects are behind the differences between the second and third row. In general equilibrium, increased capital supply – jointly with declined capital demand from firms – causes a reduction in the interest rate, which, ceteris paribus, dampens the increase in savings. This effect is strong enough to render the impact of higher life expectancy on asset holdings of those below age 55 negative. Thus, incentives to save more for a longer retirement spell are outweighed by the hike in the interest rate. Those above 55, however,

Figure 4.11: **Decomposition of changes in asset holdings.**



dis-save at a reduced pace in reaction to the higher life expectancy, as for them the need to finance a longer retirement spell dominates.

4.4.2 The Role of Labor Supply

In the previous experiment, aggregate hours did not increase in response to higher life expectancy, possibly suggesting that endogenous labor supply as an insurance mechanism is less relevant in the context of increased life expectancy than in the context of idiosyncratic productivity risk. However, to accurately identify the relevance of labor supply, we need to hold labor supply constant at its initial steady-state level (calibrations of both model versions are adjusted to maintain a capital-income ratio of 3 upon fixing labor supply).[27]

Table 4.4 depicts our results for the inelastic-labor-supply case. Under both wage processes, the equilibrium interest rate drops as a result of higher expected longevity by a greater extent than in the

[27] Again, the re-calibration is done to ensure a fair comparison in the sense that average life-time wage volatility is the same in the model versions with and without a flexible labor choice. For a more detailed explanation, see Flodén (2006).

Table 4.4: **Changes in aggregate variables with inelastic labor supply.**

	Benchmark model			**Canonical model**		
	2000	2050	Δ	2000**	2050	Δ
r	6.000%	5.442%	-0.558%	6.000%	5.523%	-0.477%
w	1.187	1.219	0.032	1.187	1.215	0.028
p	0.236	0.210	-0.026	0.236	0.210	-0.026
Capital	1.945	2.033	0.088	1.948	2.016	0.068
Hours	0.349	0.339	-0.010	0.350	0.340	-0.010
Consumption	0.501	0.497	-0.004	0.502	0.500	-0.002
Output	0.648	0.646	-0.002	0.649	0.644	-0.005
Capital/Output	3.000*	3.146	0.0146	3.000*	3.124	0.124
Welfare	-73.616	-81.050	-7.434	-73.440	-80.815	- 7.375
Coefficient of var.	0.973	0.982	0.009	0.983	0.981	-0.002

*Calibration target.

case with endogenous labor supply (compare table 4.3). This reflects the precautionary role of labor supply in insuring against an increase in life expectancy and confirms He et al. (2019)'s results: Without a flexible labor supply choice, households are missing one margin of adjustment, and thus have to build up more precautionary savings to self-insure. While the two wage processes only led to modest differences in the adjustment of the equilibrium interest rate with endogenous labor supply, the wage process becomes more relevant in this regard when labor supply is fixed. Now the interest rate drops by 8 basis points more under age-variant $\sigma^2_{\zeta_j}$ than under constant $\sigma^2_{\zeta_j}$, compared to a 0.4-basis-point difference under flexible labor supply. This highlights the particular importance of precautionary labor supply when there is age-varying idiosyncratic risk, which is also reflected by lower welfare levels in both steady states. While the identification of the theoretical mechanisms behind these findings are beyond the scope of this paper, interpretation through the lens of Low (2005)'s model suggests that with age-variant wage risk, the possibility to adjust labor supply is particularly valuable to adjust to age-varying uncertainty. The exercise suggests that the canonical wage process with constant $\sigma^2_{\zeta_j}$ leads to a significant underestimation of the role of precautionary labor supply in hedging against increased longevity risk, relative to a model with an empirically more plausible wage process.

4.5 Broader Demographic Change

The population aging observed in many advanced economies is, in most cases, not only driven by increasing life expectancy, but also by declining population growth rates. For the US, the UN projects a drop in the population growth rate from 1.1% in 2000 to 0.5% by 2050. In the following, we combine the increase in life expectancy with the projected slowdown in the population growth rate and ask if this broader definition of demographic change amplifies the differences between the two processes.

Table 4.5: **Changes in aggregate variables under broader demographic change.**

	Benchmark model			Canonical model		
	2000	2050	Δ	2000	2050	Δ
r	6.000%	5.210%	-0.790%	6.000%	5.116%	-0.884%
w	1.187	1.234	0.047	1.187	1.240	0.053
p	0.236	0.182	-0.054	0.237	0.184	-0.053
Capital	1.945	2.064	0.119	1.952	2.100	0.148
Hours	0.349	0.333	-0.016	0.351	0.334	-0.017
Consumption	0.502	0.506	0.004	0.502	0.506	0.004
Output	0.648	0.643	-0.005	0.651	0.649	-0.002
Capital/Output	3.000*	3.212	0.212	3.000*	3.239	0.239
Welfare	-71.508	-85.626	-14.118	-71.660	-86.016	-14.356
Coefficient of var.	1.046	0.997	-0.049	1.058	1.013	-0.045

*Calibration target.

4.5.1 Adjustment with a Constant Retirement Age

This subsection analyzes the effects of demographic change when the retirement age is held constant, which is the case in many advanced economies. The dotted line in figure 4.10 shows the age structure in the terminal steady state of this exercise, when both life expectancy and the population growth rates are on their projected 2050 values. A comparison to the age structure when only life expectancy has changed (the terminal steady state of the previous exercise, depicted in dashed lines) shows that accounting for the slowdown in population growth shifts mass more decisively towards elderly households.

Table 4.5 has the same structure as table 4.3 but depicts the long-run response of key variables to the joint change in both life expectancy and the population growth rate. The more extreme aging of the population reinforces the differences in the adjustment between the two models, especially for the natural interest rate. In the benchmark model, the natural interest rate drops by 79 basis points, almost ten basis points less than in the canonical model. The reason is that higher population weights of middle-aged and older households make their individual adjustment more relevant in the aggregate, and it is those households for whose adjustment the wage process makes a significant difference: Figure 4.11 shows that the life-cycle effect for households above 40 is stronger in the canonical model, i.e., those households increase saving (or decrease dis-saving) by more to prepare for a longer retirement when $\sigma^2_{\zeta_j}$ is constant. This difference across the two processes has a greater impact in the aggregate when the society is older in general, as it is the case after a drop in the population growth rate. Higher total precautionary savings in the canonical model translate into a stronger reduction of the natural interest rate. The aging of the society also amplifies the reduction in aggregate hours, which in turn implies a stronger increase in the aggregate wage level. With a significantly higher old-age dependency ratio, retirement benefits and welfare are reduced.

Table 4.6: **Changes in the interest rate with increased retirement age.**

Benchmark model				**Canonical model**			
2000	2050	Δ	$\Delta - \Delta_{\text{w/o incr.}}$	2000	2050	Δ	$\Delta - \Delta_{\text{w/o incr.}}$
6.000%	5.600%	-0.400%	0.390%	6.000%	5.572%	-0.428%	0.456%

4.5.2 Increase in Retirement Age

So far, we assumed that the retirement age stays constant when life expectancy increases on average by almost seven years, which would in reality strain public finances and give rise to political pressures for pension reform. Here we study the case in which the increase in longevity and the decline in population growth are accompanied by an increase in the retirement age that holds the old-age dependency ratio constant. While raising the retirement age in line with growing longevity proves politically difficult in many countries, a better understanding of such reforms is helpful to inform policymakers.

To stabilize the old-age dependency ratio in the terminal steady state, the retirement age has to increase by 4 years, i.e., $J_R = 70$. Table 4.6 shows the effects on the equilibrium interest rate: Columns "2000" and "2050" depict values of the natural interest rates in the initial and the terminal steady state, respectively, and column "Δ" shows the implied change. Column $\Delta - \Delta_{\text{w/o incr.}}$ reports the difference in the change in the interest rate with and without an increase in the retirement age. In both models the increase in the retirement age shortens the expected time in retirement and weakens the incentive for workers to build precautionary savings, which in turn dampens the decrease in the interest rate, relative to the case with constant retirement age. There are important differences in the size of this effect across the two models. In the benchmark model, raising the retirement age absorbs 39 basis points of the fall in the equilibrium real interest rate, while it cushions its fall by about 46 basis points under the canonical model (columns 4 and 8 respectively). Wage income at the ages of 66 to 69 – i.e., during the additional working years – is more uncertain when idiosyncratic wage risk has a u-shaped age pattern than when it is constant. As a result, raising the retirement age then causes a weaker reduction in aggregate savings and the interest rate drops by less.

4.6 Conclusion

This paper studies the response of capital and labor supply to an increase in life expectancy, taking into account the empirically documented u-shaped life-time profile of idiosyncratic wage risk. To this end, we incorporate a wage process with age-dependent wage risk into an otherwise standard OLG life-cycle model with a flexible labor supply choice à la Conesa et al. (1999), which we compare to the canonical model featuring a wage process with age-independent risk. We find that the canonical model overestimates the role of precautionary savings to self-insure against increased life expectancy, while it underestimates the role of precautionary labor supply. Depending on the age structure of the economy, this has important implications for the adjustment of aggregate variables, foremost the natural interest rate. With the canonical process, the decrease in the equilibrium interest rate as a consequence of increased life expectancy and reduced population growth is overestimated by 10 basis points. The canonical model also overestimates the impact of an increase in the retirement age in dampening the fall of the natural rate.

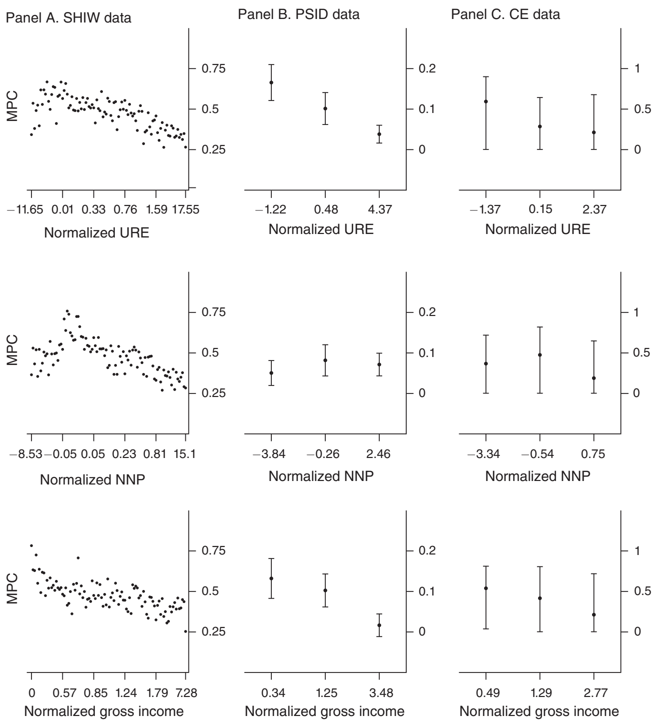
Our results show that accounting for a realistic life-time risk profile has important implications for both individual and aggregate outcomes. While the government sector of our model is very stylized, future research should focus on a more realistic modelling of the pension system. For example, a flexible retirement entry age would make it possible to study the trade-off between working more years while facing a higher risk of negative wage shocks. A further avenue for future research is to link social security benefits to a measure of past earnings, which will interact with a higher income risk at the end of the working life. Our results also show the importance of insurance through flexible labor supply. Including more realistic frictions in the labor market can be expected to diminish that role.

Appendices

A.1 Appendix to Chapter 2

A.1.1 Empirical Evidence on MPC

Figure A.1.1: **MPC and redistribution channels.**



Source: Auclert (2019).

Note: Unhedged interest rate exposure (URE) is a savings measure which, following Auclert (2019) "*measures the total resource flow that a household i needs to invest over the first period of his consumption plan.*" The net nominal position (NNP) is defined as "*difference between directly held nominal assets (mainly deposits and bonds) and directly held nominal liabilities (mainly mortgages and consumer credit).*" He uses "*pre-tax income in the PSID and the CE where it is available; in the SHIW I use post-tax income*" as the income measure..

A.1.2 Rewriting Firm i's Nominal Costs

From equation (2.7), firm i's total costs amount to:

$$C_t(i) = W_t^1 H_t^1(i) + W_t^2 H_t^2(i) + ... + W_t^N H_t^N(i). \tag{A.1}$$

Given that intermediate firms optimally allocate between the segments, we can write:

$$C_t(i) = W_t^1 \omega_1 \left(\frac{W_t^1}{W_t}\right)^{-\varepsilon_m} N_t(i) + W_t^2 \omega_2 \left(\frac{W_t^2}{W_t}\right)^{-\varepsilon_m} N_t(i) + ... + W_t^N \omega_N \left(\frac{W_t^N}{W_t}\right)^{-\varepsilon_m} N_t(i). \tag{A.2}$$

Isolating $N_t(i)$ and $W_t^{\varepsilon_m}$:

$$= \left[\omega_1 \left(W_t^1\right)^{1-\varepsilon_m} + \omega_2 \left(W_t^2\right)^{1-\varepsilon_m} + ... + \omega_N \left(W_t^N\right)^{1-\varepsilon_m}\right] W_t^{\varepsilon_m} N_t(i). \tag{A.3}$$

Note that the bracket is $W_t^{1-\varepsilon_m}$ from the definition of the wage index (2.4), so:

$$C_t(i) = W_t N_t(i). \tag{A.4}$$

A.1.3 F.O.C. Price Setting

The production function is $Y_t(i) = A_t N_t(i)$, so period profit is given by $\Pi_t(i) = P_t(i) Y_t(i) - W_t \frac{Y_t(i)}{A_t}$. Inserting the demand constraint gives:

$$\Pi_t(i) = P_t(i)^{1-\varepsilon} P_t^{\varepsilon} Y_t(i) - \frac{W_t}{A_t} P_t(i)^{-\varepsilon} P_t^{\varepsilon} Y_t(i). \tag{A.5}$$

Differentiating w.r.t. $P_t(i)$: yields:

$$\frac{\partial \pi_t(i)}{\partial P_t^*} = (1-\varepsilon)\left(P_t^*\right)^{-\varepsilon} P_t^{\varepsilon} Y_t(i) + \varepsilon \left(P_t^*\right)^{-\varepsilon-1} \frac{W_t}{A_t} P_t^{\varepsilon} Y_t(i) = 0. \tag{A.6}$$

For some period t+k, in which the newly set price prevails:

$$\frac{\partial \pi_{t+k}(i)}{\partial P_t^*} = (1-\varepsilon)\left(P_t^*\right)^{-\varepsilon} P_{t+k}^{\varepsilon} Y_{t+k}(i) + \varepsilon \left(P_t^*\right)^{-\varepsilon-1} \frac{W_{t+k}}{A_{t+k}} P_{t+k}^{\varepsilon} Y_{t+k}(i) = 0. \tag{A.7}$$

Note that the first summand is equal to $Y_{t+k|k}(i)$. Dividing by $(1-\varepsilon)$ and using $\frac{\varepsilon}{1-\varepsilon} = -\frac{\varepsilon}{\varepsilon-1}$:

$$= Y_{t+k|t}(i) - \frac{\varepsilon}{\varepsilon-1}\left(P_t^*\right)^{-\varepsilon-1} \frac{W_{t+k}}{A_{t+k}} P_{t+k}^{\varepsilon} Y_{t+k}(i) = 0. \tag{A.8}$$

Expanding by $\frac{P_t^*}{P_t^*}$ in the second term and again substituting $Y_{t+k|t}(i)$:

$$= Y_{t+k|t}(i) - \frac{\varepsilon}{\varepsilon-1}\left(P_t^*\right)^{-1} \frac{W_{t+k}}{A_{t+k}} Y_{t+k|t}(i) = 0. \tag{A.9}$$

Writing this expression in terms of nominal marginal costs $MC_t = \frac{W_t}{A_t}$:

$$= Y_{t+k|t}(i)\left[P_t^* - \frac{\varepsilon}{\varepsilon-1} MC_{t+k|t}\right] = 0, \tag{A.10}$$

so the intertemporal FOC is given by equation (2.9).

A.1.4 F.O.C. Wage Setting

The FOC for wage setting is of the following general form:

$$E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \frac{\partial U(c^n_{t+k|t}, h^n_{t+k|t})}{\partial w^{n*}_t} = 0. \tag{A.11}$$

Using the total differential and chain rule:

$$\frac{\delta U\left(c^n_{t+k|t}, h^n_{t+k|t}\right)}{\delta w^{n*}_t} = \frac{\delta U\left(c^n_{t+k|t}, h^n_{t+k|t}\right)}{\delta c^n_{t+k|t}} \frac{\delta c^n_{t+k|t}}{\delta w^{n*}_t} + \frac{\delta U\left(c^n_{t+k|t}, h^n_{t+k|t}\right)}{\delta h^n_{t+k|t}} \frac{\delta h^n_{t+k|t}}{\delta w^{n*}_t}. \tag{A.12}$$

I will compute the single components separately:

1. $\frac{\delta U\left(c^n_{t+k|t}, h^n_{t+k|t}\right)}{\delta c^n_{t+k|t}} \cong \frac{\delta U\left(c^n_{t+k}, h^n_{t+k}\right)}{\delta c^n_{t+k}} = \left(c^n_{t+k}\right)^{-\sigma}.$

2. $\frac{\delta c^n_{t+k|t}}{\delta w^{n*}_t} = \frac{\delta\left[w^{n,*}_t h^n_{t+k|t}/P_{t+k} + remainder\right]}{\delta w^{n*}_t}$, using the budget constraint. Using the labor demand $h^n_{t+k|t} = \frac{\omega_n}{\gamma_n}\left(\frac{w^n_t}{W^n_{t+k}}\right)^{-\varepsilon_n}\left(\frac{W^n_{t+k}}{W_{t+k}}\right)^{-\varepsilon_m} N_{t+k}$,
we get $\frac{\delta c^n_{t+k|t}}{\delta w^{n*}_t} = \frac{\delta\left[w^{n,*}_t \frac{\omega_n}{\gamma_n}\left(\frac{w^{n*}_t}{W^n_t}\right)^{-\varepsilon_n}\left(\frac{W^n_t}{W_t}\right)^{-\varepsilon_m} N_{t+k}/P_{t+k} + remainder\right]}{\delta w^{n*}_t}$,
so $\frac{\delta c^n_{t+k|t}}{\delta w^{n*}_t} = (1-\varepsilon_n)\frac{\omega_n}{\gamma_n}\left(w^{n,*}_t\right)^{-\varepsilon_n}\left(W^n_{t+k}\right)^{\varepsilon_n-\varepsilon_m}\left(W_{t+k}\right)^{\varepsilon_m} N_{t+k}/P_{t+k}$.

3. $\frac{\delta U\left(c^n_{t+k|t}, h^n_{t+k|t}\right)}{\delta h^n_{t+k|t}} \cong \frac{\delta U\left(c^n_{t+k}, h^n_{t+k}\right)}{\delta h^n_{t+k}} = -\left(h^n_{t+k}\right)^{\phi}.$

4. 4. $\frac{\delta h^n_{t+k|t}}{\delta w^{n*}_t} = \frac{\delta\left[\frac{\omega_n}{\gamma_n}\left(\frac{w^{n*}_t}{W^n_{t+k}}\right)^{-\varepsilon_n}\left(\frac{W^n_{t+k}}{W_{t+k}}\right)^{-\varepsilon_m} N_{t+k}\right]}{\delta w^{n*}_t} = \frac{\omega_n}{\gamma_n}(-\varepsilon_n)\left(w^{n*}_t\right)^{-\varepsilon_n-1}\left(W^n_{t+k}\right)^{\varepsilon_n}\left(\frac{W^n_{t+k}}{W_{t+k}}\right)^{-\varepsilon_m} N_{t+k}.$

Substitute these components into (A.12) to get

$$E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left[\frac{(1-\varepsilon_n)\,\omega_n}{\gamma_n}\left(c^n_{t+k}\right)^{-\sigma}\left(w^{n,*}_t\right)^{-\varepsilon_n}\left(W^n_{t+k}\right)^{\varepsilon_n-\varepsilon_m}\left(W_{t+k}\right)^{\varepsilon_m}\frac{N_{t+k}}{P_{t+k}}\right] \tag{A.13}$$

$$= E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left[\frac{\varepsilon_n\omega_n}{\gamma_n}\left(h^n_{t+k}\right)^{\phi}\left(w^{n*}_t\right)^{-\varepsilon_n-1}\left(W^n_{t+k}\right)^{-\varepsilon_m+\varepsilon_n}\left(W_{t+k}\right)^{\varepsilon_m} N_{t+k}\right] \tag{A.14}$$

Rearranging yields:

$$E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left[\left(c^n_{t+k}\right)^{-\sigma}\left(w^{n,*}_t\right)^{-\varepsilon_n}\left(W^n_{t+k}\right)^{\varepsilon_n-\varepsilon_m}\left(W_{t+k}\right)^{\varepsilon_m}\frac{N_{t+k}}{P_{t+k}}\right] \tag{A.15}$$

$$= E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left[\frac{\varepsilon_n}{1-\varepsilon_n}\left(h^n_{t+k}\right)^{\phi}\left(w^{n*}_t\right)^{-\varepsilon_n-1}\left(W^n_{t+k}\right)^{-\varepsilon_m+\varepsilon_n}\left(W_{t+k}\right)^{\varepsilon_m} N_{t+k}\right] \tag{A.16}$$

Next, write everything in relative variables, so define $W^r_t = \frac{W_t}{P_t}$, and $W^{n,r}_t = \frac{W^n_t}{P_t}$.

$$E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k N_{t+k}\left(W^{n,r}_{t+k}\right)^{\varepsilon_n-\varepsilon_m}\left(W^r_{t+k}\right)^{\varepsilon_m}\left[\left(c^n_{t+k}\right)^{-\sigma}\left(\frac{w^{n*}_t}{P_{t+k}}\right)\right] \tag{A.17}$$

$$= \frac{\varepsilon_n}{\varepsilon_n-1} E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k N_{t+k}\left(W^{n,r}_{t+k}\right)^{\varepsilon_n-\varepsilon_m}\left(W^r_{t+k}\right)^{\varepsilon_m}\left[\left(h^n_{t+k}\right)^{\phi}\right], \tag{A.18}$$

which can be re-arranged into equation (2.16).

A.1.5 Labor-Market Clearing

From the production function, we know that

$$\int_0^1 N_t(i)di = \int_0^1 \frac{Y_t(i)}{A_t}di. \tag{A.19}$$

Substituting the demand for good variety i:

$$= \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} \frac{Y_t}{A_t}di. \tag{A.20}$$

Putting into equation (2.5):

$$H_t^n = \omega_n \left(\frac{W_t^n}{W_t}\right)^{-\varepsilon_m} \frac{Y_t}{A_t}\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} di. \tag{A.21}$$

Defining $\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} di \equiv s_t^p$ as price dispersion and $\omega_n \left(\frac{W_t^n}{W_t}\right)^{-\varepsilon_m} \equiv g_t^n$ as a weight that accounts for the difference of segment n wage and average wage, we get equation (2.20).

Substituting (2.20) into (2.1) yields:

$$h_t^n(j) = \frac{1}{\gamma_n}\left(\frac{w_t^n}{W_t^n}\right)^{-\varepsilon_n} g_t^n s_t^P \frac{Y_t}{A_t}. \tag{A.22}$$

The second step is now to integrate over workers:

$$N_t = \int_0^{z_1} h_t^1(j)dj + \int_{z_1}^{z_2} h_t^2(j)dj + ... + \int_{z_{N-1}}^1 h_t^N(j)dj. \tag{A.23}$$

Substituting the result from before gives:

$$N_t = \int_0^{z_1} \frac{1}{\gamma_1}\left(\frac{w_t^1}{W_t^1}\right)^{-\varepsilon_n} g_t^1 s_t^P \frac{Y_t}{A_t}dj + ... + \int_{z_{N-1}}^1 \frac{1}{\gamma_N}\left(\frac{w_t^N}{W_t^N}\right)^{-\varepsilon_n} g_t^N s_t^P \frac{Y_t}{A_t}dj. \tag{A.24}$$

Factoring out $s_t^P \frac{Y_t}{A_t}$:

$$N_t = s_t^P \frac{Y_t}{A_t}\left[\frac{1}{\gamma_1}g_t^1 \int_0^{z_1} \left(\frac{w_t^1}{W_t^1}\right)^{-\varepsilon_n} dj + ... + \frac{1}{\gamma_N}g_t^N \int_{z_{N-1}}^1 \left(\frac{w_t^N}{W_t^N}\right)^{-\varepsilon_n} dj\right] \tag{A.25}$$

Defining $\frac{1}{\gamma_n}\int_{z_{n-1}}^{z_n} \left(\frac{w_t^n}{W_t^n}\right)^{-\varepsilon_n} dj \equiv s_t^{w,n}$, which is wage dispersion within segment n, we get equation (2.22).

A.2 Appendix to Chapter 3

In the following, I derive some additional variables of interest, following Chapter 8 of Uribe and Schmitt-Grohé (2017).

A.2.1 Real Exchange Rate

The real exchange rate is defined as[28]

$$RER \equiv \frac{\mathcal{E}P^*}{P}, \tag{A.26}$$

where P and P^* denote domestic-currency and foreign-currency price of the final good, respectively. Dividing by P^T yields

$$RER = \frac{\mathcal{E}P^*/P^T}{P/P^T}. \tag{A.27}$$

From the Law of One Price for tradable goods, $P^T = \mathcal{E}P^{T*}$, one can derive

$$RER = \frac{\mathcal{E}P^*/\mathcal{E}P^{T*}}{P/P^T} = \frac{P^*/P^{T*}}{P/P^T} \equiv \frac{p^{c*}}{p^c}, \tag{A.28}$$

where p^c and p^{c*} are defined as the domestic-currency and foreign-currency price of the final good in terms of tradables, respectively. Normalizing $p^{c*} = 1$ yields

$$RER = \frac{1}{p^c}. \tag{A.29}$$

In order to get the domestic price of the final good in terms of tradables, p^c, we can look at the equivalent decentralized economy, i.e., an economy where the aggregation of tradable and non-tradable goods into a final consumption good is not done by households, but by a perfectly competitive final-good firm. Profit maximization of such a firm implies[29]

$$\max_{C^T, C^N} \quad \Phi = p^c A(C^T, C^N) - C^T - pC^N. \tag{A.30}$$

The FOC w.r.t. C^T reads

$$p^c A_1(C^T, C^N) = 1 \Leftrightarrow p^c = \frac{1}{A_1(C^T, C^N)}, \tag{A.31}$$

where $A_1(\dots)$ denotes the partial derivative w.r.t. the first argument. Plugging into equation (A.29) yields

$$RER = A_1(C^T, C^N). \tag{A.32}$$

Since $A(\dots)$ is homothetic of degree 1, it holds that

$$RER = A_1\left(\frac{C^T}{C^N}, 1\right). \tag{A.33}$$

[28] Time indices are omitted for convenience.

[29] Note that aggregation into the final good happens now at the aggregate level, C^T and C^N, while in the decentralized setup every households performs the aggregation w.r.t. individual quantities c^T and c^N. As all households have the same preferences, however, this is equivalent. This is not valid in the non-homothetic case anymore.

From equation (3.7), we know the demand for non-tradable goods can be expressed as $p = \frac{A_2(C^T,C^N)}{A_1(C^T,C^N)} \equiv P(\frac{C^T}{C^N})$, where $A_2(\dots)$ is the partial derivative w.r.t. the second argument. Using this definition, the real exchange rate can be expressed as

$$RER = A_1(P^{-1}(p), 1). \tag{A.34}$$

With the chosen functional forms, it holds that

$$A\left(\frac{C^T}{C^N}, 1\right) = \left[\gamma \left(\frac{C^T}{C^N}\right)^{1-\frac{1}{\varepsilon}} + (1-\gamma)\right]^{\frac{1}{1-\frac{1}{\varepsilon}}} \tag{A.35}$$

and

$$A_1\left(\frac{C^T}{C^N}, 1\right) = \left(1 - \frac{1}{\varepsilon}\right)\left[\gamma \left(\frac{C^T}{C^N}\right)^{1-\frac{1}{\varepsilon}} + (1-\gamma)\right]^{\frac{1}{1-\frac{1}{\varepsilon}}-1} \gamma \left(1-\frac{1}{\varepsilon}\right)\left(\frac{C^T}{C^N}\right)^{-\frac{1}{\epsilon}}, \tag{A.36}$$

where $\frac{C^T}{C^N} = p^{\epsilon}\left(\frac{\gamma}{1-\gamma}\right)^{\epsilon}$.

A.2.2 Balance of Payments, Current Account and Net Foreign Asset Position

Aggregating the households' budget constraint, one can derive the aggregate resource constraint or balance of payments

$$\sum_a\sum_s c_t^T(a,s)\ \psi_t(s,a) + p_t\sum_a\sum_s c_t^N(a,s)\ \psi_t(s,a) + \sum_a\sum_s a_{t+1}(a,s)\ \psi_t(s,a). \tag{A.37}$$

$$= \sum_a\sum_s y_t^T\ \psi_t(s,a) + \sum_a\sum_s a_t\ \psi_t(s,a)(1+r_t) + \sum_a\sum_s s\ \psi_t(s,a)\frac{W_t}{\mathcal{E}_t}h_t + \sum_a\sum_s \frac{\phi_t}{\mathcal{E}_t}\ \psi_t(s,a), \tag{A.38}$$

Since y_t^T and ϕ_t are the same for all household, it holds that $Y_t^T = \sum_a\sum_s y_t^T\ \psi_t(s,a) = y_t^T$ and $\Phi_t = \sum_a\sum_s \phi_t\ \psi_t(s,a) = \phi_t$. Demand for assets is zero, so if net aggregate asset are larger than zero, we have a $NFA > 0$ and vice versa, where $NFA_t = \sum_a\sum_s a_t(a,s)\ \psi_t(s,a)$. Define $C_t^T = \sum_a\sum_s c_t^T(a,s)\ \psi_t(s,a)$ and $C_t^N = \sum_a\sum_s c_t^N(a,s)\ \psi_t(s,a)$. Plugging in yields

$$C_t^T + p_t C_t^N + NFA_{t+1} = Y_t^T + (1+r_t)NFA_t + \frac{\Phi_t}{\mathcal{E}_t} + \frac{W_t}{\mathcal{E}_t}\sum_a\sum_s s\ \psi_t(s,a)h_t. \tag{A.39}$$

Note that $L_t = \sum_a\sum_s s\ \psi_t(s,a)h_t$. Using the market clearing condition for non-tradable goods $C^N = Y^N$, one can derive

$$C_t^T + p_t Y_t^N + NFA_{t+1} = Y_t^T + (1+r_t)NFA_t + \frac{\Phi_t}{\mathcal{E}_t} + \frac{W_t}{\mathcal{E}_t}L_t \tag{A.40}$$

and the expression for the profit, $\Phi_t = P_t^N F(L_t) - W_t L_t$, we can derive the balance of payments in terms of tradable goods

$$C_t^T + NFA_{t+1} = NFA_t(1+r) + Y_t^T \tag{A.41}$$

$$\Leftrightarrow NFA_{t+1} = (1 + r_t)NFA_t - TB_t \tag{A.42}$$

$$\Leftrightarrow NFA_{t+1} - NFA_t = r_t NFA_t - TB_t \tag{A.43}$$

$$\Leftrightarrow CA_t = TB_t - r_t NFA_t, \tag{A.44}$$

where CA_t and TB_t denote current account and trade balance expressed in tradables, respectively. To express it in terms of the final good, divide again by p^c.

A.3 Appendix to Chapter 4

A.3.1 Computation of the Steady State

The computation of the steady state follows a standard bisection procedure, as described in Algorithm 1. We are indebted to Pavel Brendler for providing Dean Corbae's MATLAB code computing the steady state of Conesa et al. (1999). For the discretization of the non-stationary AR(1), we use Fella et al. (2019)'s non-stationary extension of the Rouwenhorst (1995) method. The corresponding MATLAB function can be found on Giulio Fella's Github.

Algorithm 1: Computation of the Steady State

1. *Make initial guesses of the steady state values of the aggregate capital stock K and aggregate labor N. Compute social security benefits p implied by these guesses.*
2. *Compute the prices w and r, which solve firm's problem.*
3. *Compute the household's decision functions by backward induction.*
4. *Compute the optimal path for savings and labor for the new born cohort by forward induction given that the initial capital stock of newborns is 0.*
5. *Compute the aggregate capital stock and aggregate labor supply.*
6. *Update K and N and return to step 2 until convergence.*

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